

ELECTROMAGNETIC INDUCTION

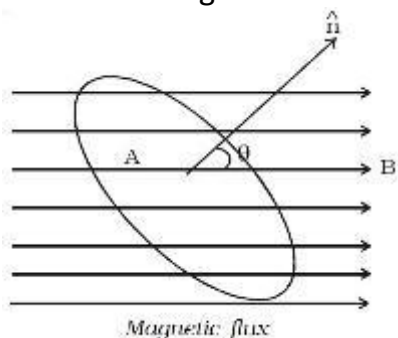
In the year 1820, Hans Christian Oersted demonstrated that a current carrying conductor is associated with a magnetic field. Thereafter, attempts were made by many to verify the reverse effect of producing an induced emf by the effect of magnetic field.

Electromagnetic induction

Michael Faraday demonstrated the reverse effect of Oersted experiment. He explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction. The discovery of this phenomenon brought about a revolution in the field of power generation.

Magnetic flux

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing a closed area (A) as shown in figure.



If θ is the angle between the direction of the field and normal to the area, then

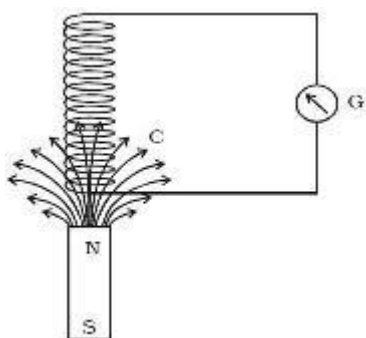
$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

Induced emf and current – Electromagnetic induction.

Whenever there is a change in the magnetic flux linked with a closed circuit an emf is produced. This emf is known as the induced emf and the current that flows in the closed circuit is called induced current. The phenomenon of producing an induced emf due to the change in the magnetic flux associated with a closed circuit is known as electromagnetic induction. Faraday discovered the electromagnetic induction by conducting several experiments.

figure consists of a cylindrical coil C made up of several turns of insulated copper wire connected in series to a sensitive galvanometer G. A strong bar magnet NS with its north pole pointing towards the coil is moved up and down. The following inferences were made by Faraday



- (i) Whenever there is a relative motion between the coil and the magnet, the galvanometer shows deflection indicating the flow of induced current.
- (ii) The deflection is momentary. It lasts so long as there is relative motion between the coil and the magnet.
- (iii) The direction of the flow of current changes if the magnet is moved towards and withdrawn from it.
- (iv) The deflection is more when the magnet is moved faster, and less when the magnet is moved slowly.
- (v) However, on reversing the magnet (i.e) south pole pointing towards the coil, same results are obtained, but current flows in the opposite direction.

Faraday demonstrated the electromagnetic induction by another experiment also.

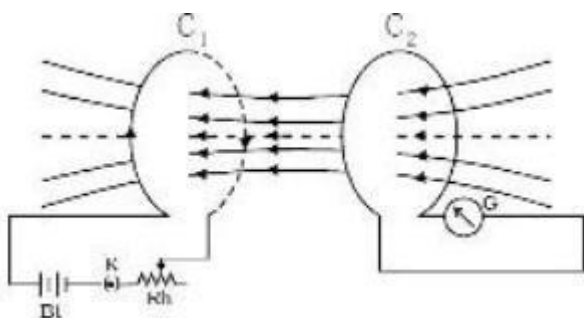


figure shows two coils C_1 and C_2 placed close to each other. The coil C_1 is connected to a battery (Bt) through a key K and a rheostat. Coil C_2 is connected to a sensitive galvanometer G and kept close to C_1 . When the key K is pressed, the galvanometer connected with the coil C_2 shows a sudden momentary deflection. This indicates that a current is induced in coil C_2 . This is because when the current in C_1 increases from zero to a certain steady value, the magnetic flux linked with the coil C_1 increases. Hence, the magnetic flux linked with the coil C_2 also increases. This causes the deflection in the galvanometer. On releasing K, the galvanometer shows deflection in the opposite direction. This indicates that a current is again induced in the coil C_2 . This is because when the current in C_1 decreases from maximum to zero value, the magnetic flux linked with the coil C_1 decreases. Hence, the magnetic flux linked with the coil C_2 also decreases. This causes the deflection in the galvanometer in the opposite direction.

Faraday's laws of electromagnetic induction

Based on his studies on the phenomenon of electromagnetic induction, Faraday proposed the following two laws.

First law

Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

Second law

The magnitude of emf induced in a closed circuit is directly proportional to the rate of change of magnetic flux linked with the circuit. Let ϕ_1 be the magnetic flux linked with the coil initially and ϕ_2 be the magnetic flux linked with the coil after a time t . Then Rate of change of magnetic flux = $(\phi_2 - \phi_1)/t$

According to Faraday's second law, the magnitude of induced emf is, $E \propto (\phi_2 - \phi_1)/t$

If $dt \rightarrow 0$, $d\phi$ is the change in magnetic flux in a time dt , then the above equation can be written as $E \propto (d\phi/dt)$

Lenz's law

The Russian scientist H.F. Lenz in 1835 discovered a simple law giving the direction of the induced current produced in a circuit. Lenz's law states that the induced current produced in a circuit always flows in such a direction that it opposes the change or cause that produces it.

If the coil has N number of turns and ϕ is the magnetic flux linked with each turn of the coil then, the total magnetic flux linked with the coil at any time is $N\phi$

$$E = -\frac{d}{dt}(N\phi)$$

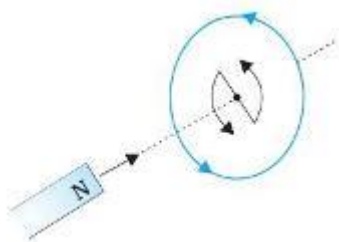
$$E = -N\frac{d\phi}{dt}$$

$$E = -\frac{N(\phi_2 - \phi_1)}{t}$$

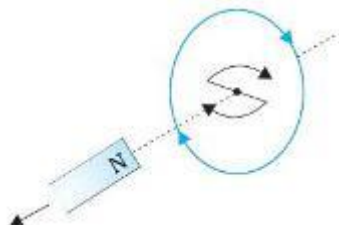
Lenz's law - a consequence of conservation of energy

Copper coils are wound on a cylindrical cardboard and the two ends of the coil are connected to a sensitive galvanometer.

A magnet is moved towards the coil as shown in figure. The upper face of the coil acquires north polarity. Consequently work has to be done to move the magnet further against the force of repulsion.



When we withdraw the magnet away from the coil, its upper face acquires south polarity. Now the work done is against the force of attraction.



When the magnet is moved, the number of magnetic lines of force linking the coil changes, which causes an induced current to flow through the coil. The direction of the induced current, according to Lenz's law is always to oppose the motion of the magnet. The work done in moving the magnet is converted into electrical energy. This energy is dissipated as heat energy in the coil.

If on the contrary, the direction of the current were to help the motion of the magnet, that is, if when north pole approaches coil south pole is produced at the face of coil due to flow of current. And when North pole goes away from coil if North pole is produced at the face of coil, magnet would start moving faster increasing the change of magnetic flux linking the coil. This results in the increase of induced current. Hence kinetic energy and electrical energy would be produced without any external work being done, but this is impossible.

Therefore, the induced current always flows in such a direction to oppose the cause. Thus it is proved that Lenz's law is the consequence of conservation of energy.

*There is another way to find the direction of current inside the loop which is described below

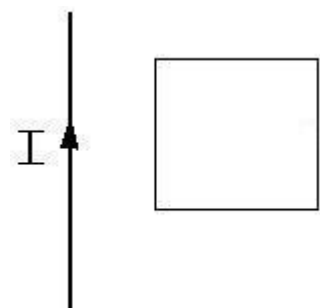


Figure shows a conducting loop placed near a long, straight wire carrying a current I as shown. The direction of magnetic field will be going inside the paper

If the current **increases** continuously, Flux linked with coil increases. Now according to Lenz's law direction of induced current will be such that it will produce magnetic field in opposite to the direction of magnetic field produced by current carrying long wire.

Thus magnetic field produced due to loop will be coming out of paper. Or direction of current will be “Anticlockwise direction”

If the current **decreases** continuously, Flux linked with coil decreases. Now according to Lenz law direction of magnetic field produced by current in loop should be along the direction of magnetic field due to wire

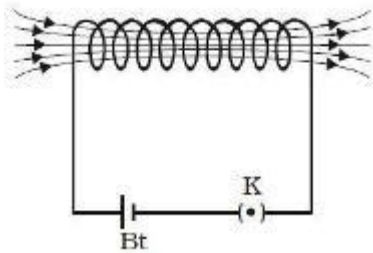
Thus current will flow in “Clockwise direction”

Fleming’s right hand rule

The forefinger, the middle finger and the thumb of the right hand are held in the three mutually perpendicular directions. If the forefinger points along the direction of the magnetic field and the thumb is along the direction of motion of the conductor, then the middle finger points in the direction of the induced current. This rule is also called generator rule.

Self Induction

The property of a coil which enables to produce an opposing induced emf in it when the current in the coil changes is called self induction. A coil is connected in series with a battery and a key (K) as shown in figure.



On pressing the key, the current through the coil increases to a maximum value and correspondingly the magnetic flux linked with the coil also increases. An induced current flows through the coil which according to Lenz’s law opposes the further growth of current in the coil.

On releasing the key, the current through the coil decreases to a zero value and the magnetic flux linked with the coil also decreases.

According to Lenz’s law, the induced current will oppose the decay of current in the coil.

Coefficient of self induction

When a current I flows through a coil, the magnetic flux (ϕ) linked with the coil is proportional to the current.

$$\phi \propto I \text{ or } \phi = LI$$

where L is a constant of proportionality and is called coefficient of self induction or self inductance.

If $I = 1A$,

$$\phi = L \times 1, \text{ then } L = \phi$$

Therefore, coefficient of self induction of a coil is numerically equal to the magnetic flux linked with a coil when unit current flows through it. According to laws of electromagnetic induction.

$$E = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt}$$

$$E = -L \frac{dI}{dt}$$

$$\text{if } \frac{dI}{dt} = 1 \text{As}^{-1}$$

$$L = -E$$

The coefficient of self induction of a coil is numerically equal to the opposing emf induced in the coil when the rate of change of current through the coil is unity. The unit of self inductance is henry (H).

One henry is defined as the self-inductance of a coil in which a change in current of one ampere per second produces an opposing emf of one volt.

Self inductance of a long solenoid

Let us consider a solenoid of N turns with length l and area of cross section A. It carries a current I. If B is the magnetic field at any point inside the solenoid, then

$$B = \frac{\mu_0 NI}{l}$$

Magnetic flux per turn = B × area of each turn

Thus Magnetic flux per turn =

$$\phi = \frac{\mu_0 NI}{l} A$$

Hence, the total magnetic flux (Φ) linked with the solenoid is given by the product of flux through each turn and the total number of turns.

$$\phi = \frac{\mu_0 NI}{l} AN$$

$$\phi = \frac{\mu_0 N^2 LA}{l}$$

If L is the coefficient of self induction of the solenoid, then

We know that $\phi = LI$

From above equations

$$LI = \frac{\mu_0 N^2 LA}{l}$$

$$L = \frac{\mu_0 N^2 A}{l}$$

If the core is filled with a magnetic material of permeability μ , then

$$L = \frac{\mu N^2 A}{l}$$

Energy associated with an inductor

Whenever current flows through a coil, the self-inductance opposes the growth of the current. Hence, some work has to be done by external agencies in establishing the current. If $\mathcal{E}$ is the induced emf then,

$$\mathcal{E} = -L \frac{dI}{dt}$$

The small amount of work dw done in a time interval dt is

$$dw = \mathcal{E} I dt$$

$$dw = -L \frac{dI}{dt} I dt$$

The total work done when the current increases from 0 to maximum value (I_0) is

$$w = \int dw = \int_0^{I_0} -LI dI$$

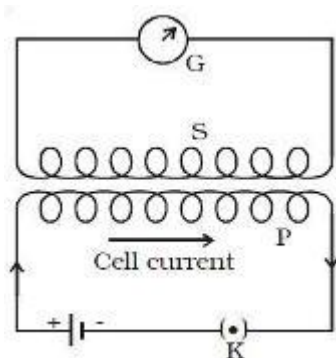
$$w = -\frac{1}{2} LI_0^2$$

This work done is stored as magnetic potential energy in the coil

Energy stored in the coil =

$$U = -\frac{1}{2} LI_0^2$$

Mutual induction



Whenever there is a change in the magnetic flux linked with a coil, there is also a change of flux linked with the neighbouring coil, producing an induced emf in the second coil.

This phenomenon of producing an induced emf in a coil due to the change in current in the other coil is known as mutual induction.

P and S are two coils placed close to each other as shown in figure, P is connected to a battery through a key K. S is connected to a galvanometer G. On pressing K, current in P starts increasing from zero to a maximum value. As the flow of current increases, the magnetic flux linked with P increases. Therefore, magnetic flux linked with S also increases producing an induced emf in S. Now, the galvanometer shows the deflection.

According to Lenz's law the induced current in S would oppose the increase in current in P by flowing in a direction opposite to the current in P, thus delaying the growth of current to the maximum value.

When the key 'K' is released, current starts decreasing from maximum to zero value, consequently magnetic flux linked with P decreases. Therefore magnetic flux linked with S also decreases and hence, an emf is induced in S. According to Lenz's law, the induced current in S flows in such a direction so as to oppose the decrease in current in P thus prolonging the decay of current.

Coefficient of mutual induction

I_P is the current in coil P and ϕ_S is the magnetic flux linked with coil S due to the current in coil P.

$$\therefore \phi_S \propto I_P \text{ or } \phi_S = M I_P$$

where M is a constant of proportionality and is called the coefficient of mutual induction or mutual inductance between the two coils.

If $I_P = 1\text{A}$, then, $M = \phi_S$

Thus, coefficient of mutual induction of two coils is numerically equal to the magnetic flux linked with one coil when unit current flows through the neighboring coil.

If E_S is the induced emf in the coil (S) at any instant of time, then from the laws of electromagnetic induction,

$$E_S = -\frac{d\phi_S}{dt} = -\frac{d(MI_P)}{dt} = -M \frac{dI_P}{dt}$$

If $(dI_P/dt) = 1$ then $E_S = -M$

Thus, the coefficient of mutual induction of two coils is numerically equal to the emf induced in one coil when the rate of change of current through the other coil is unity.

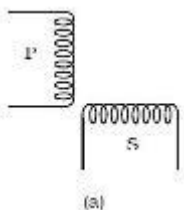
The unit of coefficient of mutual induction is henry.

One henry is defined as the coefficient of mutual induction between a pair of coils when a change of current of one ampere per second in one coil produces an induced emf of one volt in the other coil.

The coefficient of mutual induction between a pair of coils depends on the following factors

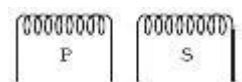
(i) Size and shape of the coils, number of turns and permeability of material on which the coils are wound.

(ii) proximity of the coils Two coils P and S have their axes perpendicular to each other as shown in figure (a) ,



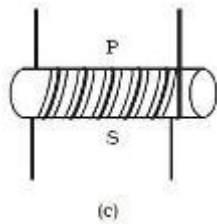
When a current is passed through coil P, the magnetic flux linked with S is small and hence, the coefficient of mutual induction between the two coils is small.

The two coils are placed in such a way that they have a common axis as shown in figure(b)



When current is passed through the coil P the magnetic flux linked with coil S is large and hence, the coefficient of mutual induction between the two coils is large.

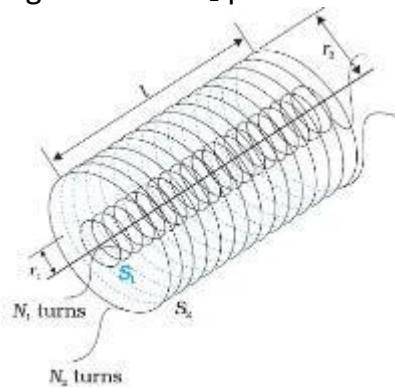
If the two coils are wound on a soft iron core as shown in figure (c)



the mutual induction is very large.

Mutual induction of two long solenoids.

S_1 and S_2 are two long solenoids each of length L . The solenoid S_2 is wound closely over the solenoid S_1 as shown in figure. N_1 and N_2 are the number of turns in the solenoids S_1 and S_2 respectively. Both the solenoids are considered to have the same area of cross section A as they are closely wound together. I_1 is the current flowing through the solenoid S_1 . The magnetic field B_1 produced at any point inside the solenoid S_1 due to the current I_1 is



$$B_1 = \mu_0 \frac{N_1}{L} I_1 \text{ ----eq(1)}$$

The magnetic flux linked with each turn of S_2 is equal to $B_1 A$.

Total magnetic flux linked with solenoid S_2 having N_2 turns is

$$\phi_2 = B_1 A N_2 \text{ ----eq(2)}$$

Substituting for B_1 from equation (1)

$$M_{21} = \frac{\mu_0 N_1 N_2 A}{L}$$

$$\phi_2 = \left(\mu_0 \frac{N_1}{L} I_1 \right) A N_2$$

$$\phi_2 = \frac{\mu_0 N_1 N_2 A I_1}{L}$$

But

$$\phi_2 = M_{21} I_1$$

From above equations

$$M_{21} = \frac{\mu_0 N_1 N_2 A}{L}$$

Also from eq(1) and eq(2) $\phi_2 = M_{21}I_1$

$$\frac{d\phi_2}{dt} = M_{21} \frac{dI_1}{dt}$$

$$E_2 = -M_{21} \frac{dI_1}{dt}$$

Taking $I_1 = 1$ unit we get $\phi_2 = M_{21}$

The constant of proportionality M_{21} is termed as the mutual inductance.

Thus “The magnetic flux linked with one of the coil of system of two coils per unit current flowing through the other coil is called mutual inductance of the system”

Instead of coil S_1 if we setup current in coil S_2 , this produces magnetic flux ϕ_1 that links with coil S_1 . If we have current I_2 flowing through coil S_2 , then $\phi_1 = B_2AN_1$ and M_{12} will be

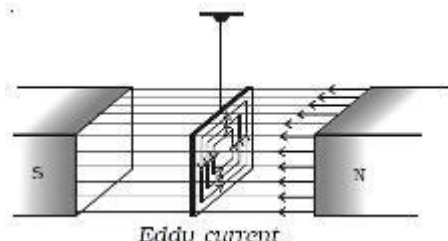
$$M_{12} = \frac{\mu_0 N_1 N_2 A}{L}$$

Unit of mutual inductance is $\text{WbA}^{-1} = \text{henry (H)}$ or VsA^{-1}

Since mutual inductance is same in both the cases $M_{21} = M_{12} = M$. this result is called reciprocity theorem.

Eddy currents

Foucault in the year 1895 observed that when a mass of metal moves in a magnetic field or when the magnetic field through a stationary mass of metal is altered, induced current is produced in the metal. This induced current flows in the metal in the form of closed loops resembling ‘eddy’ or whirl pool. Hence this current is called eddy current. The direction of the eddy current is given by Lenz’s law.



When a conductor in the form of a disc or a metallic plate as shown in Fig, swings between the poles of a magnet, electrons inside the plate experience a force $[\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})]$ because of the motion of plate. Under the influence of this force electrons move on the path which offers minimum resistance and constitute eddy currents are set up inside the plate. These currents, according to Lenz’s law, flow in such a direction that the magnetic field produced due to them opposes the motion of the conductor. Hence plates comes to rest due to damping.

If the metallic plate with holes drilled in it is made to swing inside the magnetic field, the effect of eddy current is greatly reduced consequently the plate swings freely inside the field. Eddy current can be minimized by using thin laminated sheets instead of solid metal.

Application of Eddy currents

(i) *Magnetic braking in trains*: Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents

induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth.

(ii) *Electromagnetic damping*: Certain galvanometers have a fixed core made of nonmagnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

Induction furnace: Induction furnace can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.

(iv) *Electric power meters*: The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.

You can observe the rotating shiny disc in the power meter of your house.

Methods of producing induced emf

We know that the induced emf is given by the expression

$$E = - \frac{d(NAB \cos \theta)}{dt}$$

Hence, the induced emf can be produced by changing

- (i) the magnetic induction (B)
- (ii) area enclosed by the coil (A) and
- (iii) the orientation of the coil (θ) with respect to the magnetic field.

Emf induced by changing the magnetic induction.

The magnetic induction can be changed by moving a magnet either towards or away from a coil and thus an induced emf is produced in the coil. The magnetic induction can also be changed in one coil by changing the current in the neighboring coil thus producing an induced emf.

Motional emf

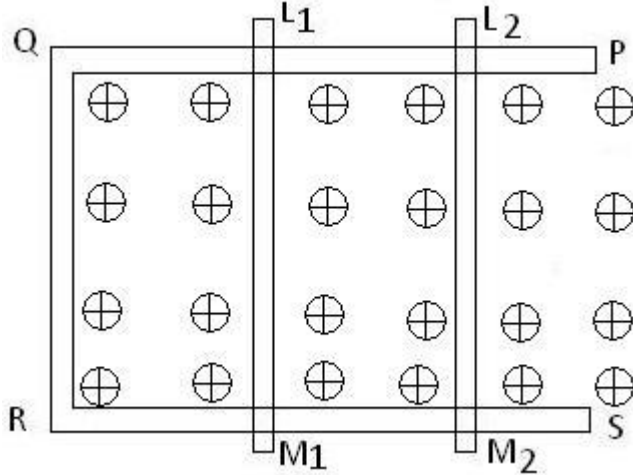
The magnetic flux $\phi = BA \cos \theta$ linked with a coil can be varied by many ways

- (1) The magnet can be moved with respect to coil
- (2) The coil can be rotated in a magnetic field (by changing angle θ between **A** and **B**)
- (3) The coil can be placed inside the magnetic field in a specific position and the magnitude of the magnetic induction B can be changed with time
- (4) The magnet can be moved inside a non-uniform magnetic field
- (5) By changing the shape of coil (that is by shrinking or stretching it)

In all above cases mentioned above, the magnetic flux linked with coil changes and hence emf is induced in the coil

“The induced emf produced in a coil due the change in magnetic flux linked with coil due to some kind of motion is called motional emf”

PQRS is a conductor bent in the shape as shown in the figure L_1M_1 is a sliding conductor of length L resting on the arms PQ and RS. A uniform magnetic field 'B' acts perpendicular to the plane of the conductor. The closed area of the conductor is L_1QRM_1 . When L_1M_1 is moved through a distance dx in time dt , the new area is L_2QRM_2 . Due to the change in area $L_2L_1M_1M_2$, there is a change in the flux linked with the conductor. Therefore, an induced emf is produced.



Change in area $dA = \text{Area } L_2L_1M_1M_2$

$$\therefore dA = Ldx$$

Change in the magnetic flux, $d\phi = B \cdot dA = BL dx$

$$E = -\frac{d\phi}{dt}$$

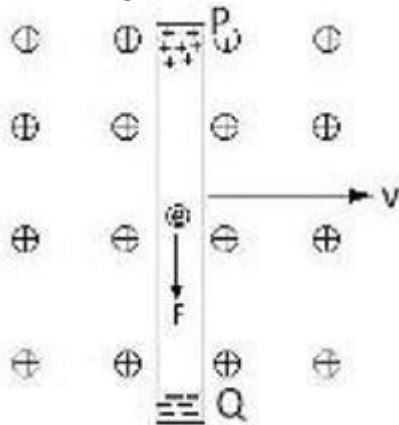
$$E = -BL \frac{dx}{dt}$$

$$E = -BLv$$

where v is the velocity with which the sliding conductor is moved.

The origin of the generation of induced emf

A conducting rod PQ is moving in a magnetic field with its plane perpendicular to it as shown in figure



The positive ions and electrons will also move along with it in the direction of motion of the rod.

In the present case, they move with a velocity $\mathbf{v}$ perpendicularly to the magnetic field $\mathbf{B}$. Hence they experience a Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. Direction of this force can be found out by using right hand screw rule which is normal to plane form by $\mathbf{v}$ and $\mathbf{B}$.

Here positive ions will experience force from Q to P but as they remains fixed at their lattice point, they will not move under the influence of this force

But electrons will flow from P to Q since electrons are free to move, they deposit at Q end of the rod and make it negatively charged.

Because of positive charge at P end of the rod and negative charge at end Q, rod behaves as battery with emf = Blv

Conversion of mechanical energy into Electrical energy

When rod moves with velocity v perpendicular to the magnetic field $\mathbf{B}$ pointing into plane of paper, emf induced is given by bvl .

Current induced in the conductor if resistance of value R is connected to the rod then current $I = E/R = Bvl/R$

Force on the conductor is given = $I (\mathbf{L} \times \mathbf{B})$

Mechanical Power $P_m = \text{Force} \times \text{velocity}$

$$P_m = BILv$$

$$P_m = BLv \left(\frac{BLv}{R} \right)$$

$$P_m = \frac{B^2 v^2 L^2}{R}$$

Electrical power generated in circuit $P_e = EI$

$$P_e = (BvL) \left(\frac{BvL}{R} \right)$$

$$P_e = \frac{B^2 v^2 L^2}{R}$$

From above equations $P_m = P_e$

Thus mechanical work done in continuing the motion of the rod is obtained in the form of electrical energy.

AC generator (Dynamo) – Single phase

The ac generator is a device used for converting mechanical energy into electrical energy.

The generator was originally designed by a Yugoslav scientist Nikola Tesla.

Principle

It is based on the principle of electromagnetic induction, according to which an emf is induced in a coil when it is rotated in a uniform magnetic field.

Essential parts of an AC generator

(i) Armature

Armature is a rectangular coil consisting of a large number of loops or turns of insulated copper wire wound over a laminated soft iron core or ring. The soft iron core not only increases the magnetic flux but also serves as a support for the coil

(ii) Field magnets

The necessary magnetic field is provided by permanent magnets in the case of low power dynamos. For high power dynamos, field is provided by electro magnet. Armature rotates between the magnetic poles such that the axis of rotation is perpendicular to the magnetic field.

(iii) Slip rings

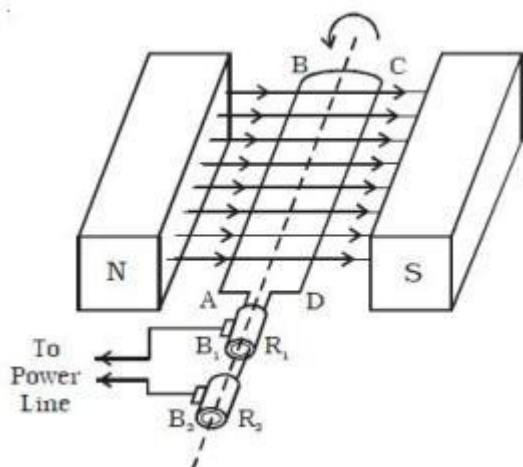
The ends of the armature coil are connected to two hollow metallic rings R_1 and R_2 called slip rings. These rings are fixed to a shaft, to which the armature is also fixed. When the shaft rotates, the slip rings along with the armature also rotate.

(iv) Brushes

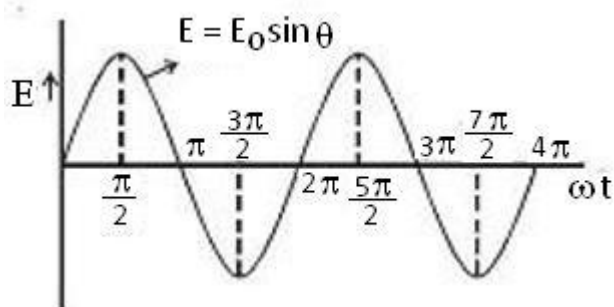
B_1 and B_2 are two flexible metallic plates or carbon brushes. They provide contact with the slip rings by keeping themselves pressed against the ring. They are used to pass on the current from the armature to the external power line through the slip rings.

Working

Whenever, there is a change in orientation of the coil, the magnetic flux linked with the coil changes, producing an induced emf in the coil. The direction of the induced current is given by Fleming's right hand rule. Suppose the armature ABCD is initially in the vertical position. It is rotated in the anticlockwise direction. The side AB of the coil moves downwards and the side DC moves upwards refer figure



Then according to Fleming's right hand rule the current induced in arm AB flows from B to A and in CD it flows from D to C. Thus the current flows along DCBA in the coil. In the external circuit the current flows from B_1 to B_2 . On further rotation, the arm AB of the coil moves upwards and DC moves downwards. Now the current in the coil flows along ABCD. In the external circuit the current flows from B_2 to B_1 . As the rotation of the coil continues, the induced current in the external circuit keeps changing its direction for every half a rotation of the coil. Hence the induced current is alternating in nature as shown in figure



As the armature completes ν rotations in one second, alternating current of frequency ν cycles per second is produced.

Flux linked with coil

$$\phi = NBA \cos \theta$$

Here θ is the angle between Area vector of coil and Magnetic field

Since coil is rotating with angular frequency ω we get $\theta = \omega t$

$$\phi = NBA \cos \omega t$$

Now

$$e = -\frac{d\phi}{dt} = -BAN \frac{d \cos \omega t}{dt}$$

$$e = BAN \omega \sin \omega t$$

Here $E_0 = BAN\omega$ is peak value of emf induced

The induced emf at any instant is given by $E = E_0 \sin \omega t$

The peak value of the emf, $E_0 = NBA\omega$ where N is the number of turns of the coil, A is the area enclosed by the coil, B is the magnetic field and ω is the angular velocity of the coil

Solved numerical

Q) A conducting circular loop of surface area $2.5 \times 10^{-3} \text{ m}^2$ is placed perpendicular to a magnetic field which varies as $B = 0.20 \sin[50\pi \text{ s}^{-1}t]$. Find the charge flowing through any cross section during the time $t = 0$ to $t = 40 \text{ s}$. Resistance of the loop is 10 Ohms

Solution:

Here $\omega = 50\pi$, $B_0 = 0.2$, $A = \text{area } 2.5 \times 10^{-3} \text{ m}^2$

Induced emf =

$$E = -\frac{d\phi}{dt}$$

$$E = -A \frac{dB}{dt}$$

$$E = -AB_0 \frac{d(\sin \omega t)}{dt}$$

$$E = -AB_0 \omega \cos \omega t$$

Induced current

$$I = \frac{E}{R} = - \frac{AB_0\omega \cos \omega t}{R}$$

$$I_0 = \frac{-AB_0\omega}{R}$$

$$I = I_0 \cos \omega t$$

The current changes sinusoidally with time period $T = 2\pi/\omega = 2\pi/50\pi = 40 \times 10^{-3} \text{ s}$

The charge flowing through any cross section during time $t = 0$ to $t = 0.04 \text{ s}$ is

$$Q = \int_0^{0.04} I dt$$

$$Q = I_0 \int_0^{0.04} \cos \omega t dt$$

$$Q = \frac{I_0}{\omega} [\sin \omega t]_0^{0.04}$$

$$Q = 0$$

Q) Two parallel, long, straight conducting conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially

A uniform magnetic field B exists at right angles to the plane formed by conductors.

Now they start moving out with a constant velocity v

- Will the induced emf depends on time
- Will the current be time dependent?

Solution:

a) Since velocity of conductor is v in outward direction displacement of each conductor is vt

Area of loop formed by the conductors will increase by $(a+2vt)^2$

Thus flux $\phi = B(a+2vt)^2$

$$E = \frac{d\phi}{dt} = 4Bv(a+2vt)$$

Thus induced emf depends on time

b) If r is the resistance per unit length then $R = 4(a+2vt)r$

Current $I =$

$$I = \frac{E}{R} = \frac{4Bv(a+2vt)}{4(a+2vt)r} = \frac{Bv}{r}$$

Thus current I is constant

Q) A solenoid of length 1 m and 0.05 m diameter has 500 turns. If a current of 2 A passes through the coil, calculate (i) the coefficient of self induction of the coil and (ii) the magnetic flux linked with the coil.

Solution:

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l}$$

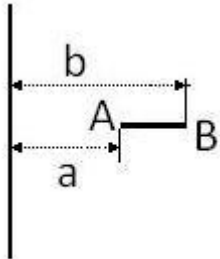
$$L = \frac{4\pi \times 10^{-7} \times (5 \times 10^2)^2 \times 3.14 (0.025)^2}{l}$$

$$L = 0.616 \text{ mH}$$

(ii) Magnetic flux $\phi = LI$

$$\phi = 0.616 \times 10^{-3} \times 2 = 1.232 \text{ milli weber}$$

Q) Figure shows a copper rod moving with velocity v parallel to a long straight wire carrying a current I . Calculate the induced emf. In the rod assuming $v = 5 \text{ m/s}$, $I = 100 \text{ amp}$, $a = 10 \text{ cm}$ $b = 20 \text{ cm}$



Solution:

The induction at a point whose perpendicular distance from rod is x is given by

$$B_{(x)} = \frac{\mu_0 I}{2\pi x}$$

The e.m.f induced on the moving the rod = E

$$E = v \int_{x=a}^{x=b} B_{(x)} dx$$

$$E = v \int_{x=a}^{x=b} \frac{\mu_0 I}{2\pi x} dx$$

$$E = \frac{\mu_0 I v}{2\pi} [\ln x]_a^b$$

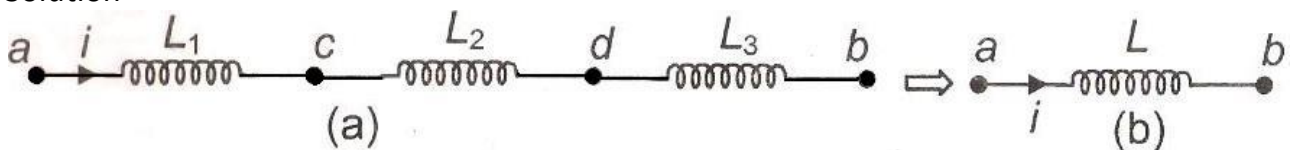
$$E = \frac{\mu_0 I v}{2\pi} \ln \left(\frac{b}{a} \right)$$

$$E = \frac{4\pi \times 10^{-7} \times 100 \times 5}{2\pi} \ln 2$$

$$E = 0.69 \times 10^{-4} \text{ V}$$

Q) Let three inductors L_1 , L_2 and L_3 are the three inductors connected in series calculate the equivalent inductance of combination

Solution



Current through each inductor is same

Let potential difference across L_1 , L_2 , L_3 be V_1 , V_2 , V_3

Thus $V = V_1 + V_2 + V_3$

Now $V_1 = L_1 (di/dt)$, $V_2 = L_2 (di/dt)$, $V_3 = L_3 (di/dt)$

Thus

$$V = L_1 \left(\frac{di}{dt} \right) + L_2 \left(\frac{di}{dt} \right) + L_3 \left(\frac{di}{dt} \right)$$

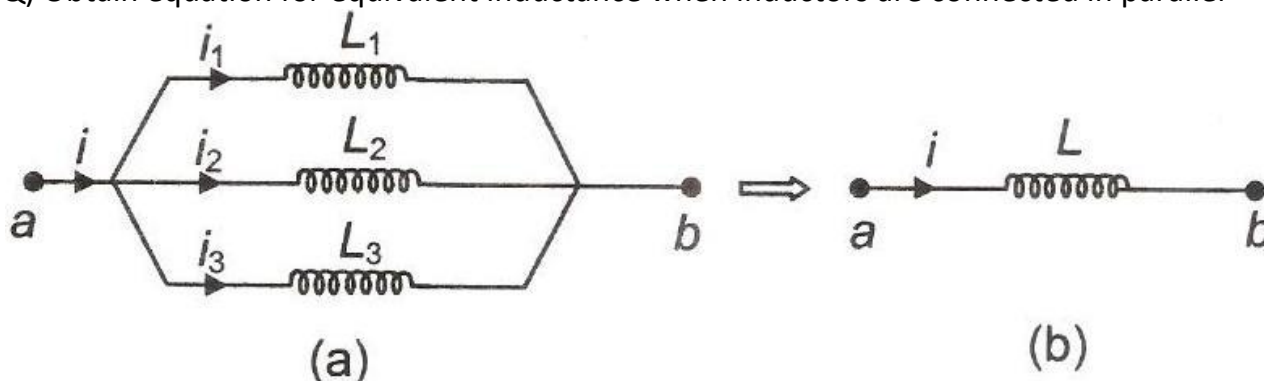
$$V = (L_1 + L_2 + L_3) \left(\frac{di}{dt} \right)$$

If equivalent inductance is L as show in figure (b)

$V = L (di/dt)$ from above equations

$L = L_1 + L_2 + L_3$

Q) Obtain equation for equivalent inductance when inductors are connected in parallel



Potential across each inductor is same

Let current through L_1 , L_2 , L_3 be I_1 , I_2 , I_3

Thus $I = I_1 + I_2 + I_3$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt} \quad \text{eq(i)}$$

Since $V = L_1 (dI_1/dt)$

$dI_1/dt = V/L_1$ similarly $dI_2/dt = V/L_2$ and $dI_3/dt = V/L_3$

Also if L is equivalent inductance then $dI/dt = V/L$

Substituting values in equation(i) we get

$$\frac{V}{L} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Questions

Q) A lamp connected in parallel with a coil of large inductance glows brilliantly before going off. Why?

Ans) This is due to large self-induced emf

Q) Can a wire act as inductor

Ans) No. This is because the magnetic flux linked with a wire of negligible cross-section area is zero

Q) A metal block and a brick of the same size area allowed to fall freely from the same height above the ground. Which of the two would reach the ground earlier and why?

Q) Two identical loops of copper and iron, are rotated with the same angular velocity in a uniform magnetic field. In which case the induced emf is more and why?

Q) A closed loop is held stationary in the magnetic field between the north and south poles of two permanent magnets held fixed. Can we hope to generate current in the loop by using very strong magnet

Q) A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0.5 second. The magnetic flux between the pole pieces is known to be 8×10^{-4} Wb. Estimate the emf induced in the wire [Ans 1.6 mV]

Q) What is the dimensional formula for mutual inductance of two coil

Q) Name the physical quantity which is measured in weber(ampere)⁻¹

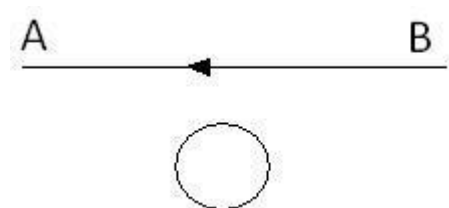
Q) Write three factors on which the self-inductance of a coil depends

Q) Why the oscillations of a copper disc in a magnetic field are damped

Q) Prove that the charge induced is independent of time

Q) Write three factors on which the mutual inductance between a pair of coils depends

Q) The electric current in a wire in the direction from B to A is decreasing. What is the direction of induced current in the metallic loop kept above the wire as shown in figure



Q) How does the self inductance of a coil change when an iron rod is introduced in the coil

Q) A ring is fixed to the wall of a room. When south pole of a magnet is brought near the ring, what shall be the direction of induced current in the ring

Q) Does change in magnetic flux induce emf or current

Ans) The induced current will be produced only if the circuit is closed. However, the induced emf will be definitely produced

Q) When a magnet falls through a vertical coil, will its acceleration be different from the 'acceleration due to gravity'?

Q) A solenoid with an iron core and a bulb are connected to a d.c. source. How does the brightness of the bulb change, when the iron core is removed from the solenoid.

Q) Why resistance coils are usually double wound?

Ans) This is done to reduce a self-inductance

Q) Two identical loops, one of copper and another of constantan, are removed from a magnetic field within the same time interval. In which loop will the induced current be greater

Q) Two identical loops, one of copper and another of constantan, are removed from a magnetic field within the same time interval. In which loop will the induced current be greater

Q) Name the physical quantity whose SI unit is weber. Is it a scalar quantity?

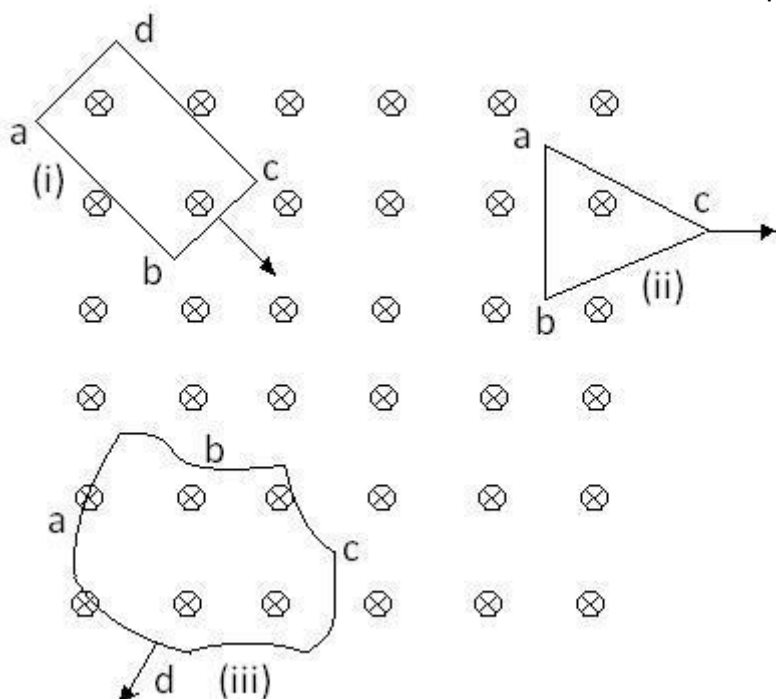
Q) An iron bar falling through the hollow region of a thick cylindrical shell made of copper experiences a retarding force. What can you conclude about the iron bar?

Q) A coil is wound on an iron core and looped back on itself so that the core has two sets of closely wound wires in series carrying current in opposite senses. What do you expect about its self inductance? Will it be large or small

Ans) The self-inductance will be small due to the cancellation of induced emf effects. This is a special example of the situation when the winding is such that $L_{eq} = L_1 + L_2 - 2M$

Or $L_{eq} = L + L - 2L = 0$

Q) Figure shows planar loops of different shapes moving out of or into a region of a magnetic field which is directed normal to the plane of the loop away from the reader. Determine the direction of induced current in each loop using Lenz's law



Ans) (i) The magnetic flux through the rectangular loop abcd increases, due to motion of the loop into region of magnetic field, The induced current must flow along the path bcdab so that it opposes the increasing flux

(ii) Due to the outward motion, magnetic flux through triangular loop abc decreases due to which the induced current flows along bacb, so as to oppose the change in flux

(iii) As the magnetic flux decreases due to motion of the irregular shaped loop abcd out of the region of magnetic field, the induced current flows along cdabc, so as to oppose change in flux

Q) Two similar circular co-axial loops carry equal currents in the same direction. If the loop be brought nearer what will happen to the current in them

Ans) When the loops are brought closer, there is an increase of magnetic flux. An induced emf has to oppose the change of magnetic flux. So the current in each loop will decrease.

Q) When a fan is switched off, a spark is produced in the switch. Why?

Ans) At the time of break of the circuit, a large emf is induced which opposes the decay of current in the circuit. This ionizes the air between the contact and of the switch.

Consequently, spark is produced

Q) Two straight and parallel wires A and B are being brought towards each other, If current in A be I , what will be the direction of induced current in B? If A and B are taken away from each other, then?

Ans) In the first case, the induced current will be opposite to the current in A. This is because of repulsion. The induced current opposes the motion of B

In the second case, the direction of induced current will be the same as the direction of current i . This is because of "attraction". The induced current opposes the motion of B

Q) How is the mutual inductance of pair of coils affected when

(i) separation between the coil is increased

(ii) the number of turns of each coil is increased

(iii) A thin iron sheet is placed between the two coils, other factors remaining the same?

Explain your answer in each case

Ans) (i) When the separation between the two coils is increased, the flux linked with the secondary due to the current in the primary decreases. Hence the mutual inductance decreases

(ii) Mutual inductance increases when the number of turns in each coil increases because $M \propto N_1 N_2$

(iii) When an iron sheet is placed between the two coils the mutual inductance increases, because $M \propto \text{permeability } (\mu)$

Q) A cylindrical bar magnet is kept along the axis of a circular coil. Will there be a current induced in the coil if the magnet is rotated about its axis

Ans) Since there is no change in magnetic flux therefore no current is induced

Q) Why birds fly off a high-tension wire when current is switched on

Ans) When current begins to increase from zero to maximum value, a current is induced in the body of the bird. This produces a repulsive force and the bird flies off

Q) Why the inductance per unit length for a solenoid near the centre is different from inductance per unit length near its end

Ans) This is because the magnetic field near the centre of the solenoid is $\mu_0 n i$. On the other hand, the magnetic field at the end is $(\mu_0 n i / 2)$

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