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TRANSFER OF HEAT

Transfer of heat

There are three ways in which heat energy may get transferred from one place to another. These are conduction, convection and radiation.

Conduction

Heat is transmitted through the solids by the process of conduction only. When one end of the solid is heated, the atoms or molecules of the solid at the hotter end becomes more strongly agitated and start vibrating with greater amplitude. The disturbance is transferred to the neighboring molecules.

Coefficient of thermal conductivity

Let us consider a metallic bar of uniform cross section A whose one end is heated. After sometime each section of the bar attains constant temperature but it is different at different sections. This is called steady state.

In this state there is no further absorption of heat.

If Δx is the distance between the two sections with a difference in temperature of ΔT and ΔQ is the amount of heat conducted in a time Δt, then it is found that the rate of conduction of heat is

(i) directly proportional to the area of cross section (A)

(ii) directly proportional to the temperature difference between the two sections (ΔT)

(iii) inversely proportional to the distance between the two sections (Δx).

$$\frac{\Delta Q}{\Delta T} \propto -A\frac{\Delta T}{\Delta x}$$

Negative sign indicates as x increases temperature decreases

$$\frac{\Delta Q}{\Delta T} = KA\frac{\Delta T}{\Delta x} \quad ----eq(1)$$

where K is a constant of proportionality called co-efficient of thermal conductivity of the metal

$\frac{\Delta T}{\Delta x}$ is called as temperature gradient

Coefficient of thermal conductivity of the material of a solid is equal to the rate of flow of heat per unit area per unit temperature gradient across the solid. Its unit is W m^{-1} K^{-1}. Or Cal $s^{-1}m^{-1}K^{-1}$

Thermal steady state

When a rod is heated , after sufficiently long time the temperature of all parts of rod become steady. These steady temperatures decreases along the length of the rod from hot end to its cold end. In this situation amount of heat energy received by the hot end in some interval is equal to the amount of heat lost by the cold end in the same time interval. Hence, any cross section of the rod, along its entire length, has the same value of heat current dQ/dt. Further along the entire length of the rod the value of the temperature gradient dT/dx is also the same along the length.

Now both dQ/dt and dT/dx remains constant with time. This condition of the rod is called 'thermal steady state" of the rod.
In thermal steady state the temperature of two ends of the rod T_1 and T_2 with $T_1>T_2$. As dT/dx is same all along the length of the rod

$$\frac{dT}{dx} = -\left[\frac{T_1 - T_2}{L}\right]$$

From eq(1)

$$\frac{dQ}{dt} = KA\left[\frac{T_1 - T_2}{L}\right] \quad ----eq(2)$$

As dQ/dt is constant

$$\frac{Q}{t} = KA\left[\frac{T_1 - T_2}{L}\right]$$

$$Q = KA\left[\frac{T_1 - T_2}{L}\right]t$$

Above equation gives the amount of heat flowing through the rod in a steady thermal state in time t.

If we represent dQ/dt as heat current (H) , which caused due to temperature difference then from equation (2)

$$H = \left[\frac{T_1 - T_2}{\frac{L}{KA}}\right]$$

Comparing the above equation with I = V/R we get thermal resistance R_H

$$R_H = \frac{L}{KA}$$

Unit of thermal resistance is Kelvin/watt and its dimensional formula is $M^{-1}L^{-2}T^{3}K$

Formula for effective thermal resistance when thermal conductors are connected in series

$$(R_H)_S = (R_H)_1 + (R_H)_2$$

For parallel connection

$$\frac{1}{(R_H)_P} = \frac{1}{(R_H)_1} + \frac{1}{(R_H)_2}$$

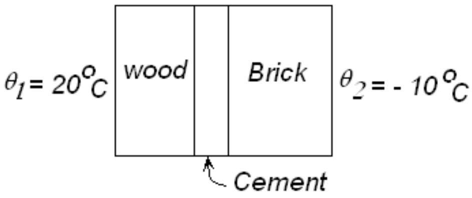
Solved Numerical

Q) An electric heater is sued in a room of total wall area 137 m^2 to maintain a temperature of $20^{O}C$ inside it, when the outside temperature is $-10^{O}C$. the walls have three different layers of materials. The innermost layer is of wood of thickness 2.5cm, the middle layer is of thickness 1.0cm and the outermost layer is of brick of thickness 25.0cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling.

The thermal conductivity of wood , cement and brick are 0.125 W/m ^{O}C, 1.5 W/m ^{O}C and 1.0 W/m ^{O}C respectively

Solution

Situation is as show in figure



The thermal resistance of wood, the cement and the brick layers are

$$R_W = \frac{L}{KA}$$

$$R_W = \frac{2.5 \times 10^{-2}}{0.125 \times 137} = \frac{0.2}{137}$$

$$R_C = \frac{1.0 \times 10^{-2}}{1.5 \times 137} = \frac{0.0067}{137}$$

$$R_B = \frac{25.0 \times 10^{-2}}{1.0 \times 137} = \frac{0.25}{137}$$

As the layers are connected in series, the equivalent

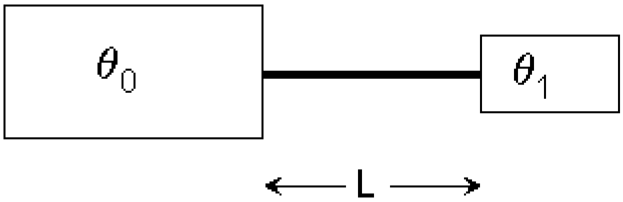
$$R = R_W + R_C + R_B$$

$$R = \frac{0.2 + 0.0067 + 0.25}{137} = 3.33 \times 10^{-3}$$

The heat current

$$i = \frac{\theta_1 - \theta_2}{R} = \frac{20 - (-10)}{3.33 \times 10^{-3}} = 9000\ W$$

Q) The figure shows a large tank of water at a constant temperature θ_0 and a small vessel containing a mass m of water at an initial temperature θ_1 ($<\theta_0$). A metal rod of length L, area of cross-section A and thermal conductivity K connects the two vessels. Find the time taken for the temperature of water in the smaller vessel to become θ_2 ($\theta_1 < \theta_2 < \theta_0$). Specific heat capacity of water is s and all other heat capacities are negligible.



Solution

Suppose, the temperature of the water in the smaller vessel is θ at time t. In the next time interval dt, a heat ΔQ is transferred from the big vessel

$$\Delta Q = \frac{KL}{A}(\theta_0 - \theta)dt \qquad ----eq(1)$$

This heat increases the temperature of the water in small tank to θ+dθ where

$$\Delta Q = ms\ d\theta \qquad ---------eq(2)$$

From equation (1) and (2)

$$ms\ d\theta = \frac{KL}{A}(\theta_0 - \theta)dt$$

$$dt = \frac{Lms}{KA}\frac{d\theta}{\theta_0 - \theta}$$

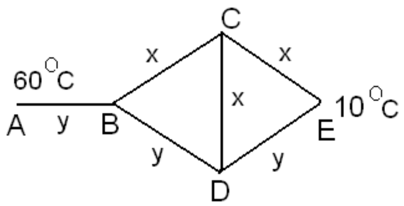
Or

$$\int_0^T dt = \frac{Lms}{KA}\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta_0 - \theta}$$

Where T is the time required for the temperature of the water to become θ_2
Thus

$$T = \frac{Lms}{KA} ln\left(\frac{d\theta}{\theta_0 - \theta}\right)$$

Q) three rods of material x and three rods of material y are connected as shown. All the rods are of identical length and cross sectional area. If the end A is maintained at 60^{O}c and the junction E is at 10^{O}C, calculate the temperature of the junctions B,C and D. The thermal conductivity of x is 0.92c.g.s unit and that of y is 0.46c.g.s units



Solution:
Since end A or rod AB is maintained at temperature higher than the end b heat is conducted from A to B
Now the total heat entering junction B is equal to the total heat leaving it (all by conduction alone)
Let the temperature of junction B, C, D be T_1, T_2 and T_3 respectively
Let the cross-sectional area of each rod be A and the length of rod be L. Then heat entering joint B per second =

$$= \frac{K_x A(60 - T_1)}{L}$$

Heat leaving B per second = heat passing through BC + heat passing through BD

$$\frac{K_x A(T_1 - T_2)}{L} + \frac{K_y A(T_1 - T_3)}{L}$$

Thus

$$\frac{K_x A(60 - T_1)}{L} = \frac{K_x A(T_1 - T_2)}{L} + \frac{K_y A(T_1 - T_3)}{L}$$

Given $K_X = 2K_Y$

$$60 - T_1 = 2\,(T_1 - T_2\,) + (T_1 - T_2\,)$$

$$\text{Or } 4T_1 - 2T_2 - T_3 = 60 \qquad \text{------eq(1)}$$

Similarly for junction c

Heat received per second = Heat passing through CD + Heat passing through CE

$$\frac{K_x A(T_1 - T_2)}{L} = \frac{K_x A(T_2 - 10)}{L} + \frac{K_x A(T_2 - T_3)}{L}$$

OR $T_1 - T_2 = T_2 - 10 + T_2 - T_3$

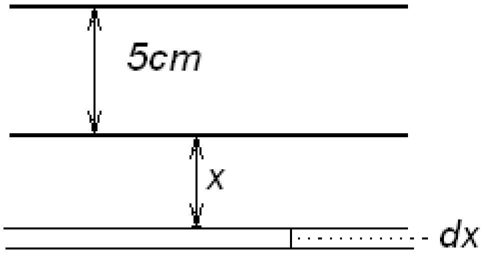
$$\text{Or } T_1 - 3T_2 + T_3 = -10 \qquad \text{----------eq(2)}$$

For Junction D

$$\frac{K_y A(T_1 - T_2)}{L} = \frac{K_x A(T_3 - T_2)}{L} + \frac{K_y A(T_3 - 10)}{L}$$

$T_1 – T_2 = 2(T3 – T_2) + T_3 – 10$
$T_1 + 2T_2 – 4T_3 = -10$ ----eq(3) Solving equation (1) (2) and (3)
we get $T_1 = 30^OC$, $T_2 = 20^O$, $T_3 = 20^OC$
Temperature of junction B = 30^OC
Temperature of junction C = 20^OC
Temperature of junction D = 20^OC

Q) The thickness of ice layer on the surface of lake is 5cm. Temperature of environment is -10^{O}C. Find the time required for the thickness of the ice layer to become double. Thermal conductivity of ice is 0.004 cal/cms ^{O}C, density of ice is 0.92 g/cm^3 and latent heat of fusion is 80cal/gm
Solution



Consider a layer of thickness dx and surface area A. Heat required to b taken out from such layer is
dQ = Adx ρ L
Here ρ is density of ice and L is latent heat of melting
If time required for passage of heat through a thickness of 5x is dt Then

$$dQ = KA\frac{\Delta T}{5+x}dt$$

Thus

$$KA\frac{\Delta T}{5+x}dt = \mathrm{Adx\ \rho\ L}$$
$$K(-10-0)dt = \mathrm{A(5+x)dx\ \rho\ L}$$

Integrating

$$-10K\int_0^t dt = \rho\ \mathrm{L}\int_0^5 (5+\mathrm{x})\mathrm{dx}$$
$$-10Kt\ =\ \rho L\left\{[5x]_0^5 + \left[\frac{x^2}{2}\right]_0^5\right\}$$

$$-10Kt\ =\ \rho L(25+12.5)$$
$$-10\times 0.004\times t\ =\ 0.92\times 80\times(37.5)$$
t= 69,000 seconds
t = 19.16 hours

Thermal expansion

The increase in dimension of a substance due to absorption of heat is called thermal expansion and decrease in dimensions of the substance by releasing the heat is called thermal contraction

Linear expansion

The increase in the length of a body with increase in temperature is called linear expansion

For small change in temperature, the increase in length $\Delta\ell$ is directly proportional to original length ℓ and increase in temperature ΔT

$\Delta\ell \propto \ell$ and $\Delta\ell \propto \Delta T$

$\Delta\ell = \alpha\, \ell$

Here α is a constant of proportionality called coefficient of linear expansion of material of the body. The value of α depends on the type of material of body and temperature. If temperature interval is very large , then α do not depends on the temperature

The unit of α is $({}^{O}C)^{-1}$ or K^{-1}

$\ell' = \ell\,(1+ \alpha\, \Delta T)$

Some substances exhibits uniform thermal expansion in all directions. Such substances are called isotropic substances. For such substance

Increase in area $\Delta A = 2\alpha A\Delta T$

Increase in volume $\Delta V= \gamma\Delta T$

$V' = V(1+ \gamma\alpha)$

For density

$P'=\rho(1+ \gamma\alpha)$

Thermal expansion is more in liquid than solid and it is maximum in gases

Solved Numerical

Q) The design of some physical instrument requires that there be a constant difference in length of 10cm between ion rod and copper rod laid side at all temperatures. Find their length ($\alpha_{Fe} = 11\times10^{-6}\ {}^{O}C^{-1}$, $\alpha_{Cu} = 17\times10^{-6}\ {}^{O}C^{-1}$)

Solution:

Since $\alpha_{Cu} > \alpha_{Fe}$ so length of iron rod should be greater than the length of cooper rod

Let initial length of iron rod be l_1 and copper rod be l_2 then

$l_1 - l_2 = 10cm$ ----------eq(1)

Also since the difference has to be constant at all the temperatures, so

$\Delta l = l_1\, \alpha_{Fe}\Delta T = l_2\, \alpha_{Cu}\, \Delta T$

$$\frac{l_1}{l_2} = \frac{\alpha_{Cu}}{\alpha_{Fe}} \qquad ---eq(2)$$

Solving equation (1) and (2), we get

$l_1 = 28.3$ cm and $l_2 = 18.3$ cm

Q) A sphere of diameter 7.0cm and mass 266.5 g floats in a bath of liquid . As a temperature is raised, the sphere begins to sink at a temperature of $35^{O}C$. If the density of the liquid is 1.527 g/cm^2 at $0^{O}C$, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere

Solution

It is given that the expansion of the sphere is negligible as compared to the expansion of liquid. At $0^{O}C$, the density of the liquid is $\rho_0 = 1.527$ g/cm^3. At $35^{O}C$, the density of the liquid equals the density of the sphere. Thus

$$\rho_{35} = \frac{266.5}{\frac{4}{3}\pi(3.5)^2} = 1.484\ g/cm^3$$

We have density ρ ∝ (1/V) thus

$$\frac{\rho_\theta}{\rho_0} = \frac{V_0}{V_\theta} = \frac{1}{(1+\gamma\theta)}$$

Or

$$\rho_\theta = \frac{\rho_0}{(1+\gamma\theta)}$$

$$\gamma = \frac{\rho_0 - \rho_{35}}{\rho_{35}(35)} = \frac{(1.527 - 1.484)}{1.484 \times 35}$$

$$\gamma = 8.28\times10^{-4}\ /\ ^{O}C$$

Convection

It is a phenomenon of transfer of heat in a fluid with the actual movement of the particles of the fluid. When a fluid is heated, the hot part expands and becomes less dense. It rises and upper colder part replaces it. This again gets heated, rises up replaced by the colder part of the fluid. This process goes on.

This mode of heat transfer is different from conduction where energy transfer takes place without the actual movement of the molecules.

Application

It plays an important role in ventilation and in heating and cooling system of the houses.

Radiation

It is the phenomenon of transfer of heat without any material medium. Such a process of heat transfer in which no material medium takes part is known as radiation.

Thermal radiation

The energy emitted by a body in the form of radiation on account of its temperature is called thermal radiation. It depends on,

(i) temperature of the body,

(ii) nature of the radiating body

The wavelength of thermal radiation ranges from 8 × 10-7 *m* to 4 × 10-4 *m*. They belong to infra-red region of the electromagnetic spectrum.

Properties of thermal radiations

1. Thermal radiations can travel through vacuum.
2. They travel along straight lines with the speed of light.
3. They can be reflected and refracted. They exhibit the phenomenon of interference and diffraction.
4. They do not heat the intervening medium through which they pass.
5. They obey inverse square law.

Absorptive and Emissive power

Absorptive power

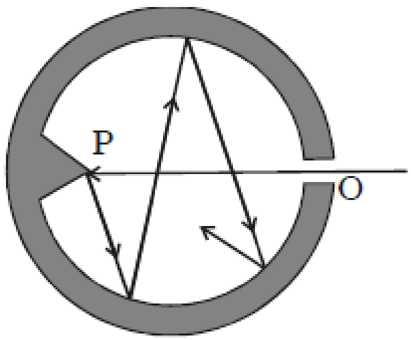
Absorptive power of a body for a given wavelength and temperature is defined as the ratio of the radiant energy absorbed per unit area per unit time to the total energy incident on it per unit area per unit time. It is denoted by a_λ.

Emissive power

Emissive power of a body at a given temperature is the amount of energy emitted per unit time per unit area of the surface for a given wavelength. It is denoted by e_λ.
Its unit is W m^{-2}.

Perfect black body

A perfect black body is the one which absorbs completely heat radiations of all wavelengths which fall on it and emits heat radiations of all wavelengths when heated. Since a perfect black body neither reflects nor transmits any radiation, the absorptive power of a perfectly black body is unity.



Fery's black body

Fery's black body consists of a double walled hollow sphere having a small opening O on one side and a conical projection P just opposite to it ref.figure Its inner surface is coated with lamp black. Any radiation entering the body through the opening O suffers multiple reflections at its inner wall and about 97% of it is absorbed by lamp black at each reflection. Therefore, after a few reflections almost entire radiation is absorbed. The projection helps in avoiding any direct reflections which even otherwise is not possible because of the small opening O. When this body is placed in a bath at fixed temperature, the heat radiations come out of the hole. The opening O thus acts as a black body radiator.

Kirchoff's Law

According to this law, the ratio of emissive power to the absorptive power corresponding to a particular wavelength and at a given temperature is always a constant for all bodies. This constant is equal to the emissive power of a perfectly black body at the same temperature and the same wavelength. Thus, if e_λ is the emissive power of a body corresponding to a wavelength λ at any given temperature, a_λ is the absorptive power of the body corresponding to the same wavelength at the same temperature and E_λ is the emissive power of a perfectly black body corresponding to the same wavelength and the same temperature, then according to Kirchoff's law

$$\frac{a_\lambda}{e_\lambda} = constant = E_\lambda$$

From the above equation it is evident that if a_λ is large, then e_λ will also be large (i.e) if a body absorbs radiation of certain wavelength strongly then it will also strongly emit the radiation of same wavelength. In other words, good absorbers of heat are good emitters also.

Applications of Kirchoff's law

(i) The silvered surface of a thermos flask is a bad absorber as well as a bad radiator. Hence, ice inside the flask does not melt quickly and hot liquids inside the flask do not cool quickly.

(ii) Sodium vapours on heating, emit two bright yellow lines. These are called D_1 and D_2 lines of sodium. When continuous white light from carbon arc passes through sodium vapour at low temperature, the continuous spectrum is absorbed at two places corresponding to the wavelengths of D_1 and D_2 lines and appear as dark lines. This is in accordance with Kirchoff's law.

Wien's displacement law

Wien's displacement law states that the wavelength of the radiation corresponding to the maximum energy (λ_m) decreases as the temperature T of the body increases.

(i.e) $\lambda_m T = b$ *where b is called Wien's constant. Its value is* 2.898×10^{-3} *m K*

Stefan's law

Stefan's law states that the total amount of heat energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of its absolute temperature.

(i.e) $E \propto T^4$ or $E = \sigma e_\lambda T^4$

where σ is called the Stefan's constant. Its value is 5.67×10^{-8} W m^{-2} K^{-4}.

It is also called Stefan - Boltzmann law, as Boltzmann gave a theoretical proof of the result given by Stefan.

If the body of temperature T is kept in an environment with temperature T_s ($T > T_S$), then rate at which the body loses heat is given by

$$\frac{dQ}{dt} = e_\lambda \sigma A(T^4 - T_S^4)$$

A : is area of surface , $e_\lambda = 1$ for perfectly black body

Solved numerical

Q) From 1 m^2 area of surface of Sun 6.3×10^7 J energy is emitted per second $\sigma = 5.669\times10^{-8}$ Wm^{-2}K^{-4}. Find the temperature of the surface of the sun

Solution.

A = 1 m^2 , dQ/dt = 6.3×10^7 J/s, $e_\lambda = 1$

$$\frac{dQ}{dt} = e_\lambda \sigma A T^4$$

$6.3\times10^7 = 5.669\times10^{-8}\times1\times1\times T^4$

T = 5841 K

Q) How many times faster the temperature of a cup of tea will decrease by 1^{O}C at 373K, then at 303K? Consider tea as a black body. Take room temperature as 293K

($\sigma = 5.7\times10^{-8}$ W m^{-2} K^{-4})
Solution

$$\frac{dQ_1}{dt} = e_\lambda \sigma A(373^4 - 293^4)$$
$$\frac{dQ_2}{dt} = e_\lambda \sigma A(303^4 - 293^4)$$

Take the ratio of above equations we get cup at 373K will decrease its temperature 11.32 times faster than cup at 303K temperature

Newton's law of cooling

Newton's law of cooling states that *the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings.*
The law holds good only for a small difference of temperature. Loss of heat by radiation depends on the nature of the surface and the area of the exposed surface.
We know that the amount of heat required to change the temperature of a bosy of mass m and specific heat c, by ΔT is
ΔQ = mcΔT
Therefore, the rate of loss of heat

$$\frac{dQ}{dt} = -mc\frac{dT}{dt}$$

According to Newton's law, the rate of loss of heat by a body depends on the difference of temperature ($T - T_S$) between body and its surrounding

$$\therefore \frac{dQ}{dt} = -mc\frac{dT}{dt} \propto (T - T_S)$$
$$\therefore \frac{dQ}{dt} = -k'(T - T_S)$$

Here dT/dt is the rate of decrease in temperature of a body at temperature T.
The constant k' depends on the mass and the specific heat of the cooling body. Negative sign indicates temperature of body decreases with time
Note that Newton's law of cooling is true only for small interval of diierence of temperature between the body and its surrounding
If the amount of heat lost by the body due to radiation is very small, this law hold true for large interval of temperature also.
For natural convection, the law of cooling given by Langmuir – Lorenz is as under

$$-\frac{dT}{dt} \propto (T - T_S)^{\frac{5}{4}}$$

Solved Numerical

Q) A body at 80^{O}C cools down to 64^{O}C in 5 minutes and in 10 minutes it cools down to 52^{O}C. What will be its temperature after 20 minutes? What is the temperature of the environment
Solution
For the first 5 minute

$\Delta T = T_2 - T_1 = 64 - 80 = -16$ and $\Delta t = 5$

$$\therefore \frac{+16}{5} = k'\left(\frac{80+64}{2} - T_s\right) \qquad ----eq(1)$$

Here we have taken the average of initial and final temperatures as the temperature of the body

Similarly for next 5 minutes

$\Delta T = 52-64 = -12$

$$\therefore \frac{+12}{5} = k'\left(\frac{52+64}{2} - T_s\right) \qquad ----eq(2)$$

Dividing eq(1) by eq(2) we get

$$\frac{16}{5} \times \frac{5}{12} = \frac{72 - T_S}{58 - T_S}$$

$232 - 4T_S = 216 - 3T_S$

$232 - 216 = T_S$

$T_S = 16^O C$

Substituting value of T_S in equation(1)

$$\frac{16}{5} = k'(72 - 16)$$

K' = 2/35

Now for third stage $\Delta t = 10$ minutes

$\Delta T = T - 52$, where T is final temperature

$$\frac{52 - T}{10} = \frac{2}{35}\left(\frac{52+T}{2} - 16\right)$$

On solving we get

$T = 36^O C$