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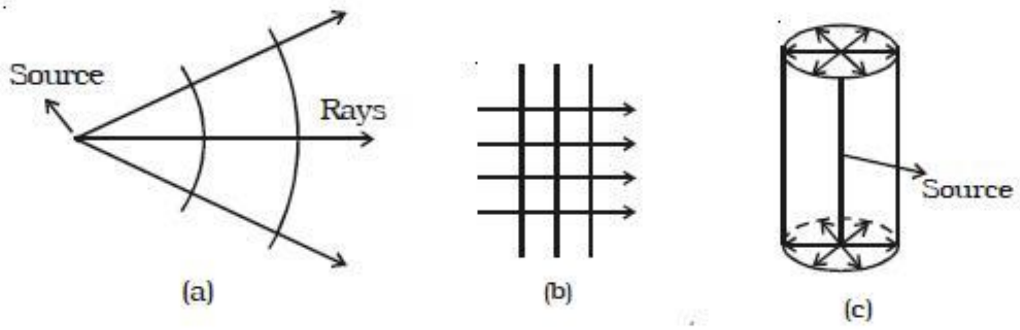
Wave Optics

Wave front

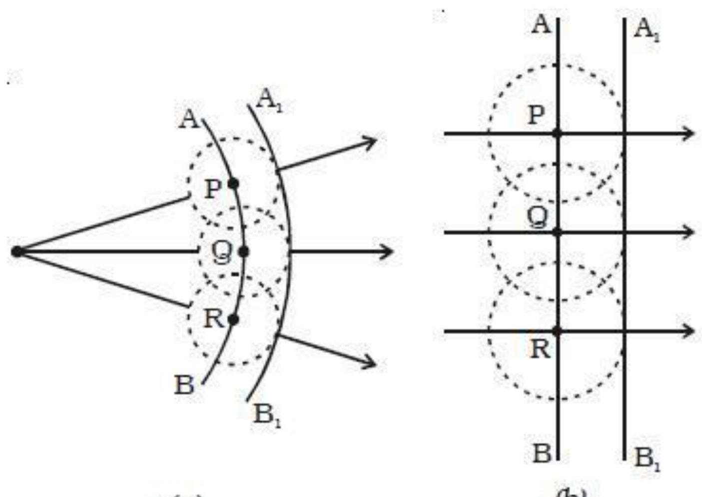
The wave front at any instant is defined as the locus of all the particles of the medium which are in the same state of vibration.
Or
Aan imaginary surface passing through particles oscillating with same phase is known as wavefront
A point source of light at a finite distance in an isotropic medium emits a spherical wave front (Fig a).
A point source of light in an isotropic medium at infinite distance will give rise to plane wavefront (Fig. b).
A linear source of light such as a slit illuminated by a lamp, will give rise to cylindrical wavefront (Fig c).



HUYGENS PRINCIPLE
Huygen's principle states that,
(i) every point on a given wave front may be considered as a source of secondary wavelets which spread out with the speed of light in that medium and
(ii) the new wavefront is the forward envelope of the secondary wavelets at that instant

Huygen's construction for a spherical and plane wavefront:

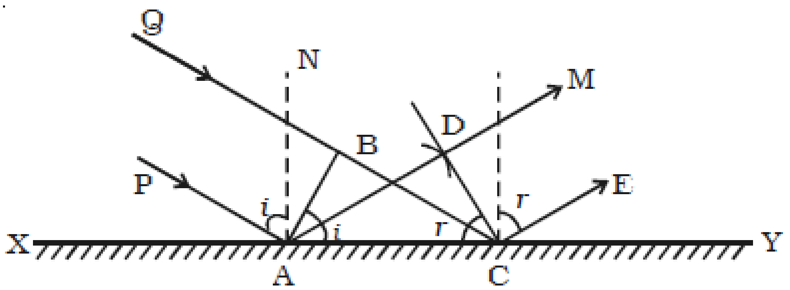
Huygen's construction for a spherical and plane wavefront is shown in Fig.a.
Let AB represent a given wavefront at a time $t = 0$. According to Huygen's principle, every point on AB acts as a source of secondary wavelets which travel with the speed of light c.
To find the position of the wave front after a time t, circles are drawn with points P, Q, R ... etc as centres on AB and radii equal to ct.
These are the traces of secondary wavelets. The arc A_1B_1 drawn as a forward envelope of the small circles is the new wavefront at that instant.
If the source of light is at a large distance, we obtain a plane wave front A_1 B_1 as shown in Fig b.



(a) (b)

Reflection of a plane wave front at a plane surface

Let XY be a plane reflecting surface and AB be a plane wavefront incident on the surface at A. PA and QBC are perpendiculars drawn to
AB at A and B respectively. Hence they represent incident rays. AN is the normal drawn to the surface.
The wave front and the surface are perpendicular to the plane of the paper (Fig.).
According to Huygen's principle each point on the wavefront acts as the source of secondary wavelet.
By the time, the secondary wavelets from B travel a distance BC, the secondary wavelets from A on the reflecting surface would travel the same distance BC after reflection.
Taking A as centre and BC as radius an arc is drawn.
From C a tangent CD is drawn to this arc. This tangent CD not only envelopes the wavelets from C and A but also the wavelets from all the points between C and A.



Therefore CD is the reflected plane wavefront and AD is the reflected ray.

Laws of reflection

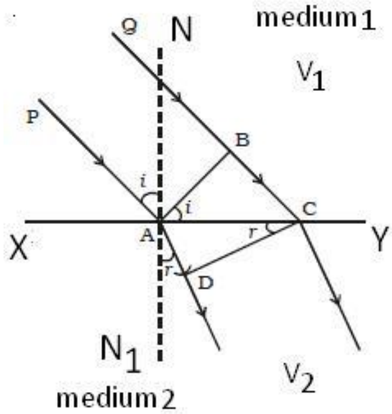
(i) The incident wavefront AB, the reflected wavefront CD and the reflecting surface XY all lie in the same plane.
(ii) Angle of incidence i = $\angle PAN = 90^0 - \angle NAB = \angle BAC$

Angle of reflection r = $\angle NAD = 90^0 - \angle DAC = \angle DCA$

$\angle B = \angle D = 90^O$
BC = AD and AC is common
∴ The two triangles are congruent
∠ BAC = ∠DCA
i.e. i = r
Thus the angle of incidence is equal to angle of reflection.

Refraction of a plane wavefront at a plane surface

Let XY be a plane refracting surface separating two media 1 and 2 of refractive indices μ_1 and μ_2 (Fig). The velocities of light in these two media are respectively v_1 and v_2.
Consider a plane wave front AB incident on the refracting surface at A. PA and QBC are perpendiculars drawn to AB at A and B respectively. Hence they represent incident rays. NAN_1 is the normal drawn to the surface. The wave front and the surface are perpendicular to the plane of the paper.
According to Huygen's principle each point on the wave front act as the source of secondary wavelet.
By the time, the secondary wavelets from B, reaches C, the secondary wavelets from the point A would travel a distance AD = v_2t, where t is the time taken by the wavelets to travel the distance BC.
∴BC = C_1t and AD = C_2t
Taking A as centre and C_2t as radius an arc is drawn in the second medium. From C a tangent CD is drawn to this arc.
.
Therefore CD is the refracted plane wavefront and AD is the refracted ray



Laws of refraction

(i) The incident wave front AB, the refracted wave front CD and the refracting surface XY all lie in the same plane.
(ii) From figure for Δ ABC and Δ ACD

$$\frac{\sin i}{\sin r} = \frac{BC/_{AC}}{AD/_{AC}} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = n_{21}$$

Constant n_{21} in above equation is known as refractive index of medium 2 with respect to medium also represented as $_1\mu_2$

This is Snell's law of refraction

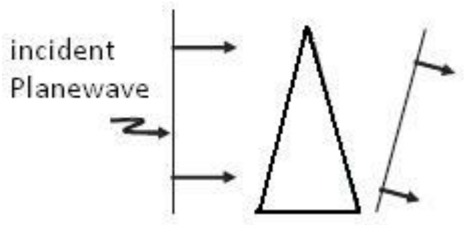
Further, if λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1 then the distance AE will be equal to λ_2 (because if the crest from B has reached C in time τ, then the crest from A should have also reached *E* in time τ); thus

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1}{v_2}$$

The above equation implies that when a wave gets refracted into a denser medium ($v_1 > v_2$) the wavelength and the speed of propagation decrease but the *frequency* f (= v/λ) *remains the same*.

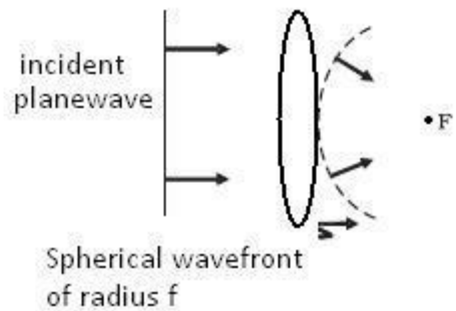
Refraction of a plane wave by a thin prism

we consider a plane wave passing through a thin prism. Clearly, since the speed of light waves is less in glass, the lower portion of the incoming wavefront (which travels through the greatest thickness of glass) will get delayed resulting in a tilt in the emerging wavefront as shown in the figure.



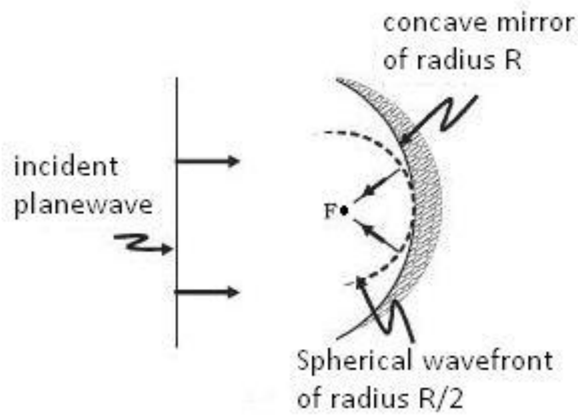
(b) a convex lens.

We consider a plane wave incident on a thin convex lens; the central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most. The emerging wavefront has a depression at the centre and therefore the wavefront becomes spherical and converges to the point F which is known as the *focus*.



(c) Reflection of a plane wave by a concave mirror

a plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focal point F.

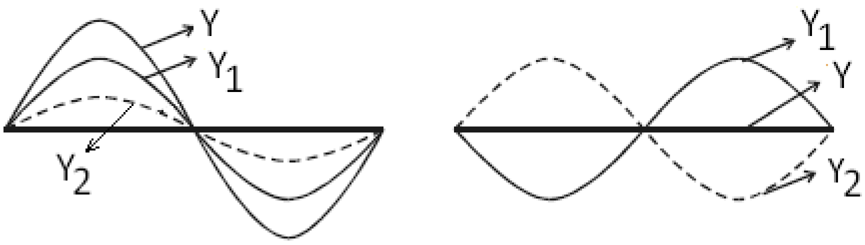


Coherent and incoherent sources

Two sources are said to be coherent if they emit light waves of the same wave length and start with same phase or have a constant phase difference.
Two independent monochromatic sources, emit waves of same wave length. But the waves are not in phase. So they are not coherent.
This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.

Superposition principle

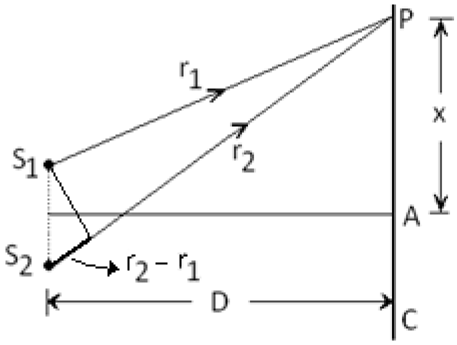
When two or more waves simultaneously pass through the same medium, each wave acts on every particle of the medium, as if the other waves are not present. The resultant displacement of any particle is the vector addition of the displacements due to the individual waves. This is known as principle of superposition. If $\mathbf{Y_1}$ and $\mathbf{Y_2}$ represent the individual displacement then the resultant displacement is given by $\mathbf{Y} = \mathbf{Y_1} + \mathbf{Y_2}$



Thus, superposition principle describes a situation when more than one waves superpose (i.e. interfere) at a point.
" The effect produced by superposition of two or more wave is called interference".

Interference due to two waves

Suppose two harmonic waves having initial phase φ_1 and φ_2 are emitted from two point like sources S_1 and S_2 respectively. They superimpose simultaneously (i.e. at the same time t) at a point P as shown in figure.



Visible perception of light is produced only by electric field, and therefore, in the present case we write light waves produced by source S_1 and S_2 in terms of electric fields (**e**) only.
Due to S_1 source,

$$\overrightarrow{e_1} = \overrightarrow{E_1} sin(\omega_1 t - k_1 r_1 + \varphi_1)$$

And due to source S_2

$$\overrightarrow{e_2} = \overrightarrow{E_2} sin(\omega_2 t - k_2 r_2 + \varphi_2)$$

Here $\mathbf{E_1}$ and $\mathbf{E_2}$ represent amplitude of electric fields, ω_1 and ω_2 denotes angular frequencies of waves, and k_1 and k_2 are wave vectors.
Let $\delta_1 = \omega_1 t - k_1 r_1 + \varphi_1$ and $\delta_2 = \omega_2 t - k_2 r_2 + \varphi_2$
Then $\mathbf{e_1} = \mathbf{E_1} \sin \delta_1$ and $\mathbf{e_2} = \mathbf{E_2} \sin \delta_2$
Now according to principle of superposition
$\mathbf{e = e_1 + e_2}$
magnitude of resultant vector **e**

$$e^2 = e_1^2 + e_2^2 + 2\overrightarrow{e_1}\overrightarrow{e_2}$$

If at a instant of time E_1 and E_2 amplitude of waves then, resultant amplitude E is

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2 cos(\delta_1 - \delta_2)$$

The average intensity of light is proportional to square of amplitude
$I \propto E^2$ thus equation becomes

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta_1 - \delta_2)\rangle$$

In above equation I_1 and I_2 are the average intensities due to each wave. They are independent of time.
The last term in above equation is known as *interference term* which depends on time
Now

$$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T}\int_{t=0}^{t=T} cos\,(\delta_1 - \delta_2)dt$$

Here t is period of electric field oscillation. On substituting value of δ_2 and δ_1 in above equation

$$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T}\int_{t=0}^{t=T} cos\,\{(\omega_1 t - \omega_2 t) + (k_2 r_2 - k_1 r_1) + (\varphi_2 - \varphi_1)\}dt \text{ --(1)}$$

Case I: Incoherent sources
If angular frequency of both source is not same thus $\cos(\delta_1 - \delta_2)$ is time dependent and average value is zero. Thus superposed two waves produce the average intensity $I_1 + I_2$ at point P
Case II: Coherent sources:
For sources to be coherent there angular frequency should be same thus $\omega_1 = \omega_2 = \omega$ (say) Also since both waves are travelling in same medium there speed will be also same thus wave length is same thus $k_1 = k_2 = k$ (say) for sake of simplicity we will consider $\varphi_2 = \varphi_1$.
From equation (1) ignoring negative sign of cos

$$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T}\int_0^T cos\{k(r_2 - r_1)\}dt$$

$$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T} cos\{k(r_2 - r_1)\}\int_0^T dt$$

$\langle cos(\delta_1 - \delta_2)\rangle = cos\{k(r_2 - r_1)\}$-- eq(2)
Further we will assume that amplitude of both waves is equal $I_1 = I_2 = I'$ then
From equation (1) and eq(2) we get

$$I = I' + I' + 2\sqrt{I'I'}cosk(r_2 - r_1)$$

$$I = 2I' + 2I'cosk(r_2 - r_1)$$

$$I = 2I'\{1 + cosk(r_2 - r_1)\}$$

$$I = 2I'\left[2cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\}\right]$$

[Use trigonometric identity $cos^2u = \frac{1+cos2u}{2}$]

$$I = 4I'cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\}$$

Here $r_2 - r_1 = \delta$ is known as the path difference between superposing waves

$$I = 4I'cos^2\left\{\frac{k\delta}{2}\right\}$$

Special Cases

Case I : Constructive Interference
For $I = 4I' = I_0$ maximum intensity of light

term $cos^2\left\{\frac{k(r_2-r_1)}{2}\right\}$ Should be equal to one, It is possible if

$$\frac{k\delta}{2} = n\pi$$

$$\frac{2\pi\delta}{2\lambda} = n\pi \ \because k = \frac{2\pi}{\lambda}$$

$$\delta = n\lambda$$

Here n = 0, 1, 2, 3, ……

"If the path difference between superposing waves is nλ (n = 0, 1, 2, 3, …..) intensity at a superposing point is maximum. Such interference is called constructive interference"

From equation

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta_1 - \delta_2)\rangle$$

For constructive interference cos(δ_2-δ_1) = 0 thus

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

Or

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{max} \propto (A_1 + A_2)^2$$

Case II: Destructive Interference

For intensity I = 0

term $cos^2\left\{\frac{k(\delta)}{2}\right\}$ Should be equal to zero, It is possible if

$$\frac{k\delta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \dots ....$$

Or

$$\frac{k\delta}{2} = (2n-1)\frac{\pi}{2}$$

As k = 2π/λ

$$\frac{2\pi\delta}{2\lambda} = (2n-1)\frac{\pi}{2}$$

$$\delta = (2n-1)\frac{\lambda}{2}$$

Here n = 1, 2, 3, 4, …..

" If phase difference between superposing waves is (2n-1)π intensity at a superposing point is minimum. This interference is called destructive interference"

"If path difference between superposing wave is (2n-1) (λ/2) intensity at superposed point is minimum. Such interference is known as destructive interference"

Here n = 1, 2, 3, 4, ….

From equation

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta_1 - \delta_2)\rangle$$

For destructive interference cos($\delta_1 - \delta_2$) = -1 thus

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Or

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$I_{min} \propto (A_1 - A_2)^2$$

Condition for sustained interference

The interference pattern in which the positions of maximum and minimum intensity of light remain fixed with time, is called sustained or permanent interference pattern. The conditions for the formation of sustained interference may be stated as :

(i) The two sources should be coherent
(ii) Two sources should be very narrow
(iii) The sources should lie very close to each other to form distinct and broad fringes

Solved numerical

Q) Two sources of intensity I and 3I are used in an interference experiment. Find the intensity at a point where the waves from the two sources superimpose with a phase difference (1) Zero (2) π/2

Solution:

In case of interference

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta_1 - \delta_2)\rangle$$
$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta)\rangle$$

(1) As δ = 0, cosδ =1

$$\therefore I' = 3I + I + 2\sqrt{(3I)(I)} \times 1$$
$$I' = 4I + 2\sqrt{3}I$$

(2) As δ = π/2, cosδ = 0

$$\therefore I' = 3I + I + 2\sqrt{(3I)(I)} \times 0$$
$$\therefore I' = 4I$$

Q) Ratio of the intensities of rays emitted from two different coherent sources is α. For the interference pattern formed by them, prove that

$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{1 + \alpha}{2\sqrt{\alpha}}$$

I_{max} : Maximum of intensity in the interference fringe
I_{min} : Minimum of intensity in the interference fringe
Solution:

Given $I_1 = \alpha I_2$
Since $I \propto A^2$
Thus $A_1 = \sqrt{\alpha}\, A_2$
Now

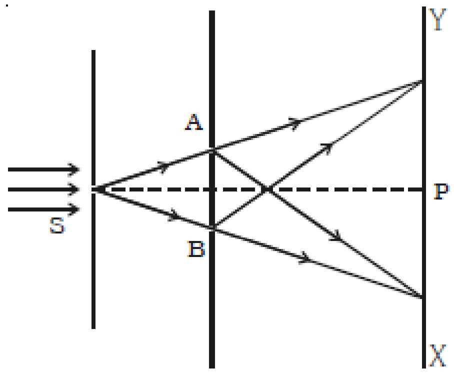
$$I_{max} \propto (A_1 + A_2)^2$$
$$I_{max} \propto \left(\sqrt{\alpha}A_2 + A_2\right)^2$$
$$I_{max} \propto A_2^2\left(\sqrt{\alpha} + 1\right)^2$$

And

$$I_{min} \propto (A_1 - A_2)^2$$
$$I_{min} \propto A_2^2\left(\sqrt{\alpha} - 1\right)^2$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{A_2^2\left(\sqrt{\alpha} + 1\right)^2 + A_2^2\left(\sqrt{\alpha} - 1\right)^2}{A_2^2\left(\sqrt{\alpha} + 1\right)^2 - A_2^2\left(\sqrt{\alpha} - 1\right)^2}$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{\left(\sqrt{\alpha} + 1\right)^2 + \left(\sqrt{\alpha} - 1\right)^2}{\left(\sqrt{\alpha} + 1\right)^2 - \left(\sqrt{\alpha} - 1\right)^2}$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{\alpha + 1 + 2\sqrt{\alpha} + \alpha + 1 - 2\sqrt{\alpha}}{\alpha + 1 + 2\sqrt{\alpha} - \alpha - 1 + 2\sqrt{\alpha}}$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{2(\alpha + 1)}{4\sqrt{\alpha}} = \frac{\alpha + 1}{2\sqrt{\alpha}}$$

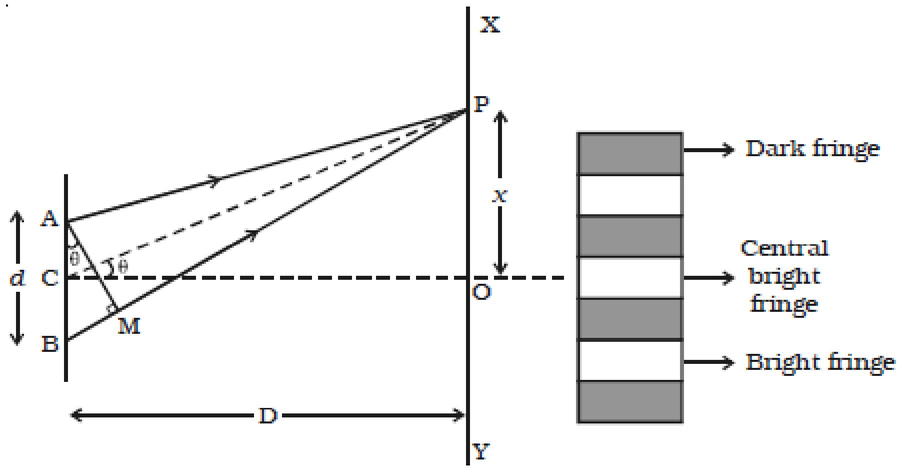
Young’s double slit experiment

The phenomenon of interference was first observed and demonstrated by Thomas Young in 1801. The experimental set up is shown in Fig.
Light from a narrow slit S, illuminated by a monochromatic source, is allowed to fall on two narrow slits A and B placed very close to each other.
The width of each slit is about 0.03 mm and they are about 0.3 mm apart. Since A and B are equidistant from S, light waves from S reach A and B in phase. So A and B act as coherent sources.
According to Huygen’s principle, wavelets from A and B spread out and overlapping takes place to the right side of AB. When a screen XY is placed at a distance of about 1 metre from the slits, equally spaced alternate bright and dark fringes appear on the screen. These are called interference fringes or bands.
Using an eyepiece the fringes can be seen directly. At P on the screen, waves from A and B travel equal distances and arrive in phase. These two waves constructively interfere and bright fringe is observed at P. This is called central bright fringe.
When one of the slits is covered, the fringes disappear and there is uniform illumination on the screen. This shows clearly that the bands are due to interference.



Expression path difference in terms of D and x

Let *d* be the distance between two coherent sources A and B of wavelength λ. A screen XY is placed parallel to AB at a distance D from the coherent sources. C is the mid point of AB. O is a point on the screen equidistant from A and B. P is a point at a distance *x* from O, as shown in Fig. Waves from A and B meet at P in phase or out of phase depending upon the path difference between two waves.



Draw AM perpendicular to BP The path difference δ = BP – AP AP = MP
∴ δ = BP – AP = BP – MP = BM
In right angled Δ ABM, BM = d sin θ
If θ is small, sin θ = θ
∴ The path difference δ = θ.d
In right angled triangle COP,

$$tan\theta = \frac{OP}{CO} = \frac{x}{D}$$

For small values of θ, tan θ = θ
∴ The path difference

$$\delta = \frac{xd}{D}$$

Location of bright and dark fringes on screen (x)

Bright fringes:

By the principle of interference, condition for constructive interference is the path difference = nλ

$$n\lambda = \frac{xd}{D}$$

$$x = \frac{n\lambda D}{d}$$

By substituting n = 0, 1, 2, 3…

If n = 0 then we get location central bright fringe

n=1, we get then location of first bright fringe

n=2, we get then location of second bright fringe

n=3, , we get then location of third bright fringe… etc

Dark fringes:

By the principle of interference, condition for destructive interference is the path difference

$$\delta = (2n-1)\frac{\lambda}{2}$$

$$(2n-1)\frac{\lambda}{2} = \frac{xd}{D}$$

$$x = \frac{D(2n-1)\lambda}{2d}$$

By substituting n = 1, 2, 3…

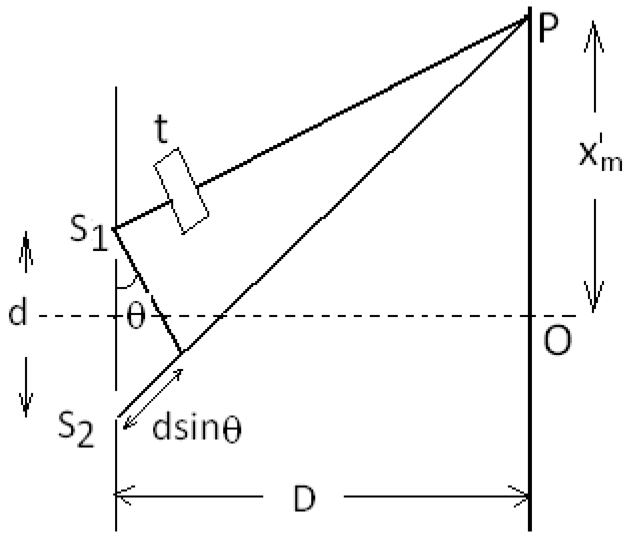
n=1, we get then location of first dark fringe

n=2, we get then location of second dark fringe

n=3, we get then location of third dark fringe… etc

Displacement of fringe pattern

If a thin transparent slab of thickness t and refractive index μ is placed in front of one of sources, for example, in front of S_1. This changes the path difference because light from S_1 now travels more optical path than earlier. *(Optical path in medium is equal to the product of refractive index of the medium to geometrical path length in air)*



Path difference before placing slab = $S_2P - S_1P = r_2 - r_1 = \delta$
On placing slab path length of thickness t effective path length $S_1P = (r_1 - t) + t\mu$
Thus effective path difference after placing slab $\delta' = r_2 - [(r_1 - t) + t\mu]$
$\therefore \delta' = (r_2 - r_1) + t(\mu - 1)$
From the geometry of figure $r_2 - r_1 = d\sin\theta$, since θ is very small

$$r_2 - r_1 = d\tan\theta = \frac{dx'_m}{D}$$

$$\therefore \delta' = \frac{dx'_m}{D} + (\mu - 1)t$$

$$x'_m = \frac{n\lambda D}{d} - (\mu - 1)\frac{tD}{d}$$

In absence of slab, the m^{th} maxima is given by

$$x_m = \frac{n\lambda D}{d}$$

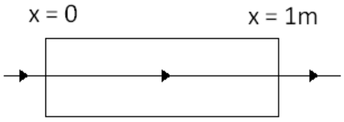
Therefore, the fringe shift is given by

$$x_0 = x_m - x'_m = (\mu - 1)\frac{tD}{d}$$

When a transparent slab is introduced, the fringe pattern shifts in the direction where the slab is placed.

Solved numerical

Q) A ray of light travels through a slab as shown in figure. The refractive index of the material of the slab varies as $\mu = 1.2 + x$, where $0 \leq x \leq 1$ m. What is the equivalent optical path of the glass slab?



Solution
Consider a small geometric path dx then optical path = μdx
Thus optical path op =

$op = \int_0^1 (1.2\ +\ \mathrm{x})dx = \left[1.2x + \frac{x^2}{2}\right]_0^1$

$$op = 1.2 + \frac{1}{2} = \frac{3.4}{2} = 1.7$$

Distance between two consecutive bight or dark fringe Or Width of fringe , Band width (β)

The distance between any two consecutive bright or dark bands is called bandwidth.
The distance between $(n+1)^{th}$ and n^{th} order consecutive bright fringes from O is given by

$$\beta\ =\ x_{n+1} - x_n = (n+1)\ \frac{\lambda D}{d} - (n)\frac{\lambda D}{d} = \frac{\lambda D}{d}$$

Similarly, it can be proved that the distance between two consecutive dark bands is also equal to $\frac{\lambda D}{d}$

Angular fringe width or angular separation between fringes is

$$\theta = \frac{\lambda}{d}$$

Since bright and dark fringes are of same width, they are equi−spaced on either side of central maximum.

Solved numerical

Q) In Young's double slit experiment, angular width of a fringe formed on a distant screen is 0.1^{O}. the wavelength of the light used is 6000Å . What is the spacing between the slit. If above setup is immersed in liquid it is observed that angular fringe width is decreased by 30% find refractive index of liquid

Solution

Angular fringe width or angular separation between fringes is

$$\theta = \frac{\lambda}{d}$$

$$d = \frac{\lambda}{\theta} = \frac{6000 \times 10^{-10}}{0.1 \times \frac{\pi}{180}} = 3.44 \times 10^{-4} m$$

(ii)Given that when set up with first light is immersed in liquid angular width decreases by 30% thus wave length of first light in liquid = 4200Å

From formula for refractive index

$$\mu = \frac{\lambda_{air}}{\lambda_{liq}} = \frac{6000}{4200} = 1.428$$

Q) In Young's double slit experiment using monochromatic light the fringe pattern shifts by certain distance on the screen when a mica sheet of refractive index 1.6and thickness 1.964 micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of the slit and the screen is doubled. It is found that the distance between successive maximum now is the same as the observed fringe shift upon the introduction of mica sheet. Calculate the wavelength of the light.

Solution

Due to introduction of mica sheet, the shift on the screen

$$x_0 = (\mu - 1)\frac{tD}{d}$$

Given that on removal of mica sheet and increasing the distance between screen and slit two times, then it is observed that distance between successive maximum now is the same as the *observed fringe shift* upon the introduction of mica sheet

Thus $x_o = \beta$ fringe width when distance is doubled

Now fringe width

$$\beta = \frac{\lambda(2D)}{d}$$

Therefore

$$(\mu - 1)\frac{tD}{d} = \frac{\lambda(2D)}{d}$$

$$\lambda = \frac{(\mu - 1)t}{2}$$

$$\lambda = \frac{(1.6 - 1)(1.964 \times 10^{-6})}{2} = 5892\text{Å}$$

Q) In Young's double slit experiment a beam of light of wavelength 6500Å and 5200Å is used . Find the minimum distance from the central bright fringe where bright fringe produced by both the wavelength get superposed. The distance between two slit is 0.5mm and the distance between the slits and the screen is 100cm.

Solution

Let n^{th} bright fringe due wavelength 6500Å to and m^{th} bright fringe due to 5200Å superposed at distance x from central bright spot

Thus $x_n = x_m$

$$\frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d}$$

$$n\lambda_1 = m\lambda_2$$

$$n6500 = m5200$$

$$\frac{m}{n} = \frac{6500}{5200} = \frac{5}{4}$$

That is 4^{th} bright fringe of 6500 Å and 3^{rd} bright fringe of 5200 Å superpose

Taking n =4 in equation we get the minimum distance from the central bright fringe where bright fringe produced by both the wavelength get superposed

$$x_n = \frac{n\lambda_1 D}{d}$$

$$x_n = \frac{4 \times 6500 \times 10^{-8} \times 100}{0.05} = 0.52cm$$

Q) The ratio of the intensities at minima to maxima in the interference pattern is 9:25. What will be the ratio of the widths of the two slits in young's double slit experiment

Solution

Intensity is proportional to width of slit, so amplitude is proportional to the square root of the width of the slit

$$\frac{A_1}{A_2} = \sqrt{\frac{\mathrm{w}_1}{\mathrm{w}_2}}$$

W_1 and w_2 represent the width of the two slits

$$\frac{I_{min}}{I_{max}} = \frac{(A_1 - A_2)^2}{(A_1 + A_2)^2}$$

$$\frac{I_{min}}{I_{max}} = \frac{\left(1 - \frac{A_2}{A_1}\right)^2}{\left(1 + \frac{A_2}{A_1}\right)^2}$$

$$\frac{9}{25} = \frac{\left(1 - \frac{A_2}{A_1}\right)^2}{\left(1 + \frac{A_2}{A_1}\right)^2}$$

$$\frac{3}{5} = \frac{1 - \frac{A_2}{A_1}}{1 + \frac{A_2}{A_1}}$$

$$8\frac{A_2}{A_1} = 2$$

$$\frac{A_1}{A_2} = \frac{4}{1}$$

Thus from equation (1)

$$\frac{4}{1} = \sqrt{\frac{\mathrm{w}_1}{\mathrm{w}_2}}$$

$$\frac{w_1}{w_2} = \frac{16}{1}$$

Condition for obtaining clear and broad interference bands

(i) The screen should be as far away from the source as possible.

(ii) The wavelength of light used must be larger.

(iii) The two coherent sources must be as close as possible.

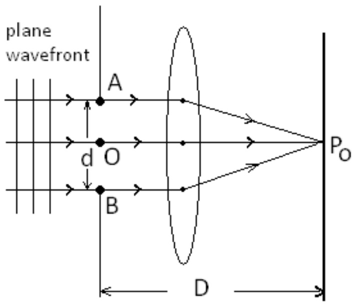
Diffraction

When waves encounters obstacles or openings like slits, they bend round the edges. This bending of wave is called diffraction. Diffraction is the effect produced by the limiting part of the wavefront.

Smaller is the width of the slit, more will be diffraction for given wavelength. It is also found that if the wavelength and the width of the slit are so changed that ratio (λ/d) remains constant, amount of bending or diffraction does not change.

If ratio λ/d is more ,then more is the diffraction

Diffraction due to single slit



Central Maxima:

Consider a plane wavefront arrive at a plane of slit, according to Huyen's principle all the point on the slit like AOB acts as secondary source having the same phase and produce secondary waves.

Those waves originated from each points of a slit and diffracted normal to the plane of the slit or we can say in the direction of incident wave will be concentrated at point P_o by lens.

In figure out of a many waves only three rays are shown in figure.

Screen is at focal length of lens .

Ray emitted from A and B are in phase and passes equal distance through air and lens thus they are in phase when get converged at P_o .

Now ray emitted from O travel less distance in air but more distance in lens, in lens velocity of light gets reduced thus optical path travelled by the ray emitted by O is equal to optical path due to ray A and B.. Thus all rays meeting at P^o are in phase produces central bright fringe.

(Optical path in medium is equal to the product of refractive index of the medium to geometrical path length in air)

First minimum

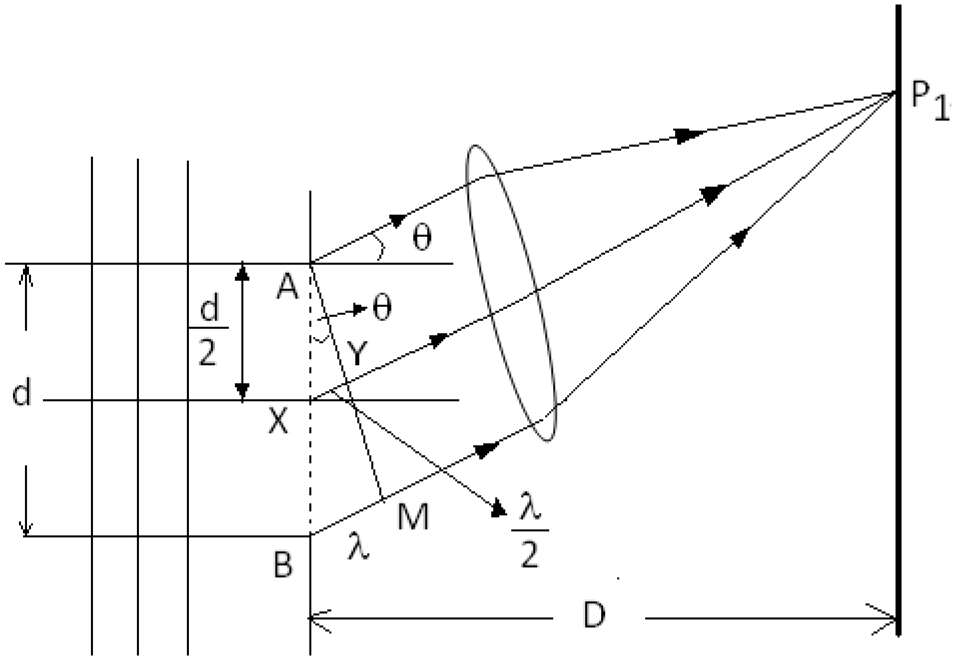
As shown in figure consider a waves which is diffracted an angle θ with respect to perpendicular bisector XP_o of the slit. Here, point X is the midpoint of slit AB. Therefore AX = Xb = d/2.

Here secondary waves originated from all points A, X, B of slit are through to be divided in two parts

Wave from AX and waves from X to B.

As per figure, all these waves diffracted at an angle θ are focused at point P_1 of a screen.

Draw AM $\perp$ BL. It is obvious that all the rays reaching from AM to P_1 have equal optical path

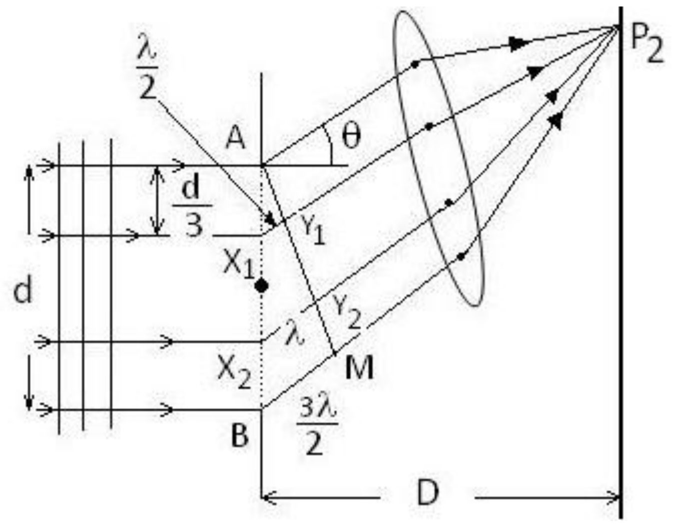


But rays going from A and X, and reaching to point P_1 have path difference of XY
Let assume diffraction angle be θ is such that XY = λ/2
In this situation, waves from A and X will follow the condition of destructive interference at point P_1 and resultant intensity will be zero
Further for all point between AX there exists a point between XB ,such that ray from point between XB have path difference of λ/2 with respect to rays from point between AX
Thus in totality, destructive interference will take place at point P_1
Point P_1 is known as first minimum

Condition for minima:

From geometry of figure $d\sin\theta = \lambda$
General equation is
$d\sin\theta = n\lambda$
For n = 1 we get first minima
n = 2 we get second minima

First Maxima:
As shown in figure suppose slit AB is assumed to be divided in three equal parts AX_1, X_1X_2, X_2B . Here $AX_1 = X_1X_2 = X_2B = d/3$.
Draw AM ⊥ BL. Wave reaching from AM to P_2 will have equal optical path
Waves starting from A and X_1 and imposing at point P_1 will have path difference X_1Y_1 .



Let us assume that diffraction θ is such that

$$X_1Y_1 = \frac{\lambda}{2}$$
$$X_2Y_2 = \lambda$$
$$BM = \frac{3\lambda}{2}$$

Since path difference between waves originated from A and X_1 and superimpose at point P_2 is λ/2, they interfere destructively. And intensity at point P_2 due to these waves will be zero.

In the same way, waves from every pair AX_1 and X_1X_2 will have path difference λ/2 and resultant intensity at point P_2 due to them is zero.

However, intensity of ray diffracted at an angle θ from section X_1B is not vanishing at point P_2. Therefore due to this section of the slit intensity at point P_2 will not be zero. And point P_2 will be bright

Here point P_2 is known as first maximum. It is obvious that intensity at point P_2 will be far less than central bright spot

Condition for minima:

From geometry of figure for fist maxima

$$dsin\theta \ = \frac{3\lambda}{2}$$

General formula

$$dsin\theta \ = \frac{(2n+1)\lambda}{2}$$

For n = 1 we get first maxima

n = 2 we get second maxima

Angular width

For first order minima dsinθ =λ or

$$sin\theta = \frac{\lambda}{d}$$

For small angle

$$\theta = \frac{\lambda}{d}$$

Also sinθ = tanθ
From geometry of figure

$$tan\theta = \frac{x}{D}$$

$$\therefore \frac{x}{D} = \frac{\lambda}{d}$$

∴ width of central maxima

$$2x = \frac{2\lambda D}{d}$$

Angular width of central maxima is given by

$$2\theta = \frac{2\lambda}{d}$$

Comparison between Interference and diffraction

(i) The interference pattern has a number of equally spaced bright and dark bands. The diffraction pattern has a central bright maximum which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre, on either side.
(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.
(iii) For a single slit of width *a*, the first null of the interference pattern occurs at an angle of λ/*a.* At the same angle of λ/*a*, we get a maximum (not a null) for two narrow slits separated by a distance *a.*

Solved numerical

Q)Angular width of central maximum in diffraction obtained by single slit using light of wavelength 6000Å is measured. If light of another wavelength is used, the angular width of the central maximum is found to be decreased by 30%. Find
(i)The other wavelength (ii) If the experiment is repeated keeping the apparatus in a liquid, the angular width of central maxima decreases by the same amount (30%0, find its refractive index
Solution:
(i)Angular fringe width or angular separation between fringes is

$$2\theta = \frac{2\lambda}{d}$$

For first light

$$\theta_1 = \frac{\lambda_1}{d}$$

For second light

$$\theta_2 = \frac{\lambda_2}{d}$$

$$\frac{\theta_2}{\theta_1} = \frac{\lambda_2}{\lambda_1}$$

But θ_2 is 70% of θ_1
That is , $\theta_2 = 0.7\ \theta_1$

$$\therefore 0.7 = \frac{\lambda_2}{\lambda_1}$$

$$\lambda_2 = 0.7 \times 6000 = 4200\ \text{Å}$$

Q) A slit of width d is illuminated by white light. For what value of d will the first minimum for red light of wavelength λ_R =6500Å appear at θ =15^O? What is the situation for violet colour having wavelength λ_V = 4333Å at the same point. Sin15^O = 0.2588
Solution:
Since the diffraction occurs separately for each wavelength, we have to check condition for minia and maxima for each wavelength separately
For the first minimum of red colour n = 1, using condition
$d\sin\theta = n\lambda_R$

$$d = \frac{n\lambda_R}{sin\theta} = \frac{1 \times 6500 \times 10^{-10}}{sin15}$$

d=2.512×10^{-6} m
For violet colour wavelength is different. We have to check whether slit width above can give us minima or maxima
Thus $d\sin\theta = n'\lambda_V$

$$n' = \frac{dsin\theta}{\lambda_v} = \frac{2.512 \times 10^{-6} \times 0.2588}{4333 \times 10^{-10}} = 1.5$$

Here n' should be integer. Thus, for violet colour condition for minima does not satisfy
Now using condition for maxima

$$dsin\theta = (2n' + 1)\frac{\lambda_V}{2}$$

$$n' = \frac{dsin\theta}{\lambda_v} - \frac{1}{2} = 1.5 - 0.5 = 1.0$$

This result suggest that for violet colour first maximum is observed

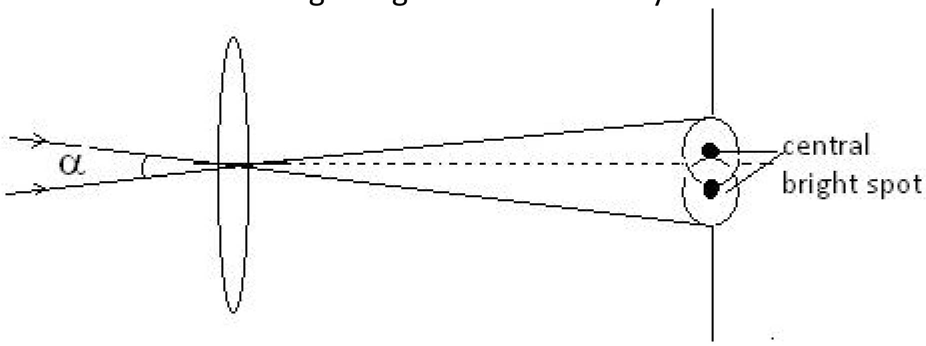
Resolving power of optical instrument

When a beam of light (light waves) from a point like object passes through the objective of an optical instruments, the lens acts like a circular aperture and produces a diffraction pattern instead of sharp point image.
If there are two point objects kept closed to each other, their diffraction pattern may overlap. Then it may be difficult to distinguish them as separate.
The criterion to get distinct and separate images of two closely placed point like objects was given by Rayleigh
" The images of two point like objects can be seen as separate if the central maximum in the diffraction pattern of one falls either on the first minimum of the diffraction pattern of the other or it is at grater separation"
For the case of circular aperture diffraction due to lens of diameter D. Rayleigh's criterion is given by

$$sin\alpha \approx \alpha = \frac{1.22\lambda}{D}$$

Resolving power of telescope:

Consider a parallel beam of light falling on a convex lens. If the lens is well corrected for aberrations, then beam will get focused to a point.
However, because of diffraction, the beam instead of getting focused to a point gets focused to a spot of finite area. In this case the effects due to diffraction can be taken into account by considering a plane wave incident on a circular aperture followed by a convex lens.
Taking into account the effects due to diffraction, the pattern on the focal plane would consist of a central bright region surrounded by concentric dark and bright rings.



If two stars are very close to each other separated by angle α will be very small and the diffraction pattern of both stars will mingle with each other. In this situation it is difficult to see both the stars distinctly and clearly

"Ability of an optical instrument to produce distinctly separate images of two closely placed objects is called its resolving power"
It is clear that for optical instruments resolving power depends on angle α . is a minimum angle to see two images distinctly

$$\alpha_{min} = \frac{1.22\lambda}{D}$$

Here D is diameter of lens and λ is wavelength
Width of the central maxima or radius is given by

$$\alpha_{min} f = \frac{1.22\lambda}{D} f$$

Here α_{min} is known as angular resolution of the telescope, while its inverse is known as resolving power or geometrical resolution
Thus resolving power of telescope

$$\frac{1}{\alpha_{min}} = \frac{D}{1.22\lambda}$$

Solved numerical

Q) Calculate the useful magnifying power of a telescope of 11cm objective. The limit of resolution of eye is 2' and wavelength of light used is 5000Å
Solution
The magnifying power of a telescope is given by
M = D/d, where D is diameter of the objective and d is diameter of eye piece
For normal (useful) magnification, diameter of eyepiece should be equal to the diameter of the pupil d_e of the eye. Therefore, useful magnification is
M = D/d_e
From the equation of limit of resolution of telescope

$$d\theta = \frac{1.22\lambda}{D}$$

$$d\theta = \frac{1.22 \times 5500 \times 10^{-10}}{11 \times 10^{-2}} = 6.1 \times 10^{-6}\, rad$$

Limit of resolution of eye is given dθ'=2'

$$d\theta' = \frac{2 \times 3.14}{60 \times 180^o} = 5.815 \times 10^{-4}\, rad$$

∴ Useful magnification

$$\frac{d\theta'}{d\theta} = \frac{5.815 \times 10^{-4}}{6.1 \times 10^{-6}} = 95.3$$

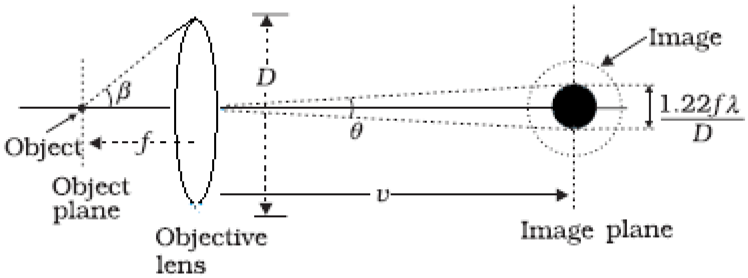
Q) Huble space telescope is at a distance 600 km from earth's surface. Diameter of its primary lens (objective) is 2.4m. When a light of 550nm is used by this telescope, at what minimum angular distance two objects can be seen separately? Also obtain linear minimum distance between these objects. Consider these objects on the surface of earth and neglect effects of atmosphere.

Solution:

$$\alpha_{min} = \frac{1.22\lambda}{D} = \frac{1.22 \times 550 \times 10^{-9}}{2.4}$$
$$\alpha_{min} = 2.8 \times 10^{-7}\, rad$$

Linear distance between objects= $\alpha_{min}L$
Where L = distance between object telescope and object
Linear distance between objects = $2.8\times10^{-7}\times600\times10^{3}$ = 0.17m

Resolving power of microscope:



Let the diameter of lens be D and its focal length be f. As object distance is usually kept greater than that of f . Let the image distance be v. the angular width of central maximum due to the effect of diffraction is ,

$$\theta = \frac{1.22\lambda}{D}$$

Width of central maximum

$$\theta v = \frac{1.22\lambda}{D} v$$

If image of two point like objects are at a separation les than vθ, then it will be seen as a mixed single object. It can be proved that a minimum distance (d_m) for which objects can be seen separately is given by

$$d_m = \theta \frac{v}{m}$$

Here m is magnification m=v/f substituting value of m we get d_m

$$d_m = \theta \frac{v}{v/f} = \theta f$$

Substituting value of θ we get

$$d_m = \frac{1.22\lambda}{D} f$$

From figure D/2 =f (tanβ)
D = 2f (tanβ) substituting value in above equation we get

$$d_m = \frac{1.22\lambda}{2f tan\beta} f = \frac{1.22\lambda}{2tan\beta}$$

For small angles tanβ = sinβ
Reciprocal of d_m known as Resolving Power(RP) of microscope

$$RP = \frac{1}{d_m} = \frac{2\sin\beta}{1.22\lambda}$$

If some medium with large refractive index (n) is used between object and objective resolving power of microscope increases n times
Formula for resolving power is given by

$$RP = \frac{2n\sin\beta}{1.22\lambda}$$

Here term nsinβ is known as 'Numerical Aperture". Resolving power is inversely proportional to wavelength.

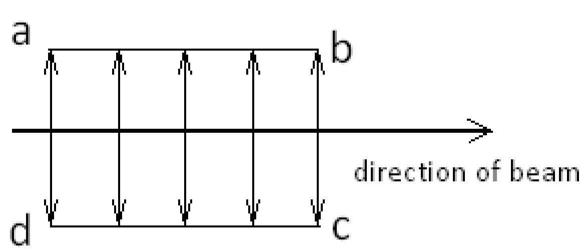
Polarization

The phenomena of reflection, refraction, interference, diffraction are common to both transverse waves and longitudinal waves. But the transverse nature of light waves is demonstrated only by the phenomenon of polarization.

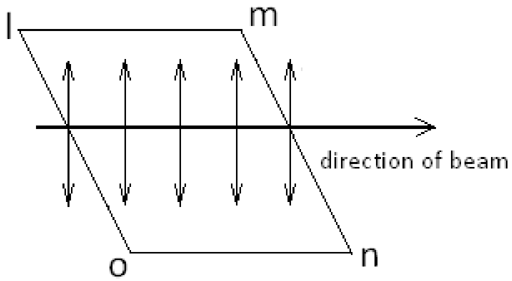
Unpolarized light
In an ordinary light source like bulb, there are large numbers of atomic emitters. They all emit electromagnetic waves with there Electrical vector E , vibrating randomly in all directions perpendicular to direction of propagation.
It means that vector E of one wave is not parallel to Vector E of another wave.
Wave emitted by different atom is of source propagate in same direction constitute beam of light.
If such beam is assumed to be coming out of paper, light vectors (E) of its waves will be found in all random direction in a plane of paper. Such light is called Unpolarized light.
" In a beam of light, if the oscillations of E vectors are in all direction in a plane perpendicular to the direction of propagation, then the light is called unpolarized light"

Polarized light

If in beam of light all electric vector (E) are coplanar and parallel to each other is plane polarized light
Process by which getting the plane polarized light from unpolarized light is called polarization



" The plane containing the direction of the beam and the direction of oscillation of E vectors is called the plane of oscillation
. In figure abcd is the plane of oscillation



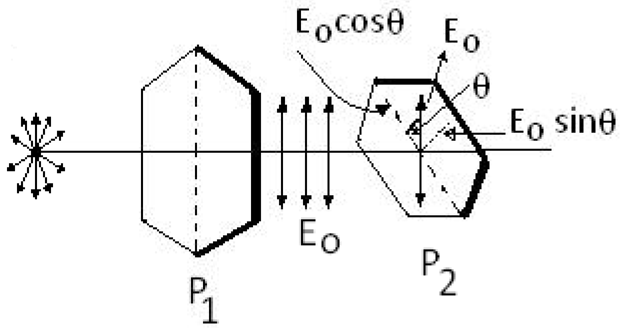
"A plane perpendicular to the plane of oscillation and passing through the beam of light is called the plane of polarization"

In figure lmno is the plane of polarization

When light passes through tourmaline crystal ,freely transmit the light components which are polarized to a definite direction. While crystal absorbs light strongly whose polarization is perpendicular to this definite direction. Thus emergent beam iof light only coplanar and parallel E vectors are found. This definite direction in a crystal is known as an *optic axis*

Malus' Law

If the light from an ordinary source (like a sodium lamp) passes through a polaroid sheet P_1, it is observed that its intensity is reduced by half. Rotating P_1 has no effect on the transmitted beam and transmitted intensity remains constant.
Now, let an identical piece of polaroid P_2 be placed before P_1. On rotating P_2 has a dramatic effect on the light coming from P_2.
In one position, the intensity transmitted by P_2 followed by P_1 is nearly zero. When turned by 90^o from this position, P_1 transmits nearly the full intensity emerging from P_2
An optic axis of plate P_2 makes an angle of θ with that of the plate P_1. In this situation vector E emerging from plate P_1 (E_0) makes angle θ with an optic axis of plate B. therefore we can resolve them into two components
91) $E_0\cos\theta$ parallel to the optic axis of plate P_2 and
(2) $E_0\sin\theta$ perpendicular to the optic axis of plate P_2



Thus, only $E_0\cos\theta$ components will emerge out of plate $P_{2'}$ while perpendicular components are absorbed. Since intensity is proportional to the square of amplitude, intensity of light incident on plate P_2 is

$$I \propto E_0^2 cos^2\theta$$

$$\therefore \frac{I}{I_0} = cos^2\theta$$

$$\therefore I = I_0 cos^2\theta$$

This equation is known as Malus Law. It is obvious from above equation that if plate P_2 is completely rotated, twice the intensity of emerging light is zero , corresponding to $\theta = \pi/2$ and $3\pi/2$ and twice it become maximum corresponding to $\theta = 0$ and $\theta = \pi$.
This procedure will help us to verify whether the given light is polarized or not. Since plate P_2 is used to analyze a state of polarization of incident light, it is known as Analyzer.

Solved numerical

Q) A ray of light travelling in water is incident on a glass plate immersed in it. What the angle of incident is 51^o the reflected ray is totally plane polarized. Find the refractive index of glass. Refractive index of water is 1.33

Solution:

Angle of incidence $\theta_P = 51^o$

Since at this incidence angle, reflected ray is totally plane polarized, using Brewster'slaw refractive index of glass w.r.t. water is

$n' = \tan\theta_P = \tan 51 = 1.235$

But refractive index n' =

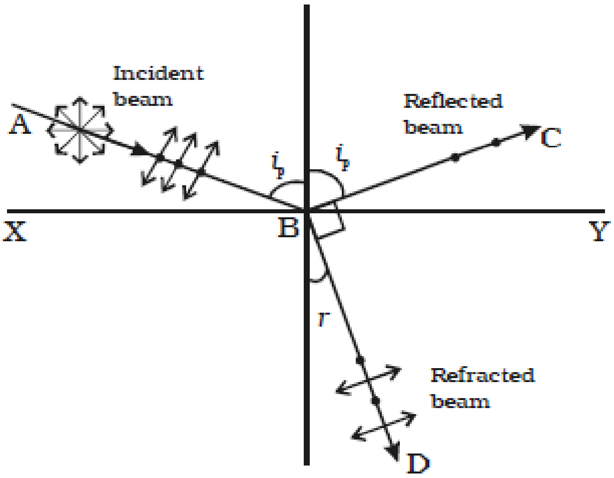
$$n' = \frac{RI.glass(n_g)}{R.i.of\ water(n_w)}$$

$n_g = n' n_w = 1.235 \times 1.33 = 1.64$

Polarisation by reflection

The simplest method of producing plane polarised light is by reflection. Malus, discovered that when a beam of ordinary light is reflected from the surface of transparent medium like glass or water, it gets polarised. The degree of polarisation varies with angle of incidence.

Consider a beam of unpolarised light AB, incident at any angle on the reflecting glass surface XY. Vibrations in AB which are parallel to the plane of the diagram are shown by arrows. The vibrations which are perpendicular to the plane of the diagram and parallel to the reflecting surface, shown by dots (Fig).



A part of the light is reflected along BC, and the rest is refracted along BD.
On examining the reflected beam with an analyzer, it is found that the ray is partially plane polarised. When the light is allowed to be incident at a particular angle, (for glass it is 57.5^o) the reflected beam is completely plane polarised. The angle of incidence at which the reflected beam is completely plane polarised is called the polarising angle (i_p).

Brewster's law

Sir David Brewster conducted a series of experiments with different reflectors and found a simple relation between the angle of polarization and the refractive index of the medium.
It has been observed experimentally that the reflected and refracted rays are at right angles to each other, when the light is incident at polarizing angle.
From Fig, ip $+90^0 + r = 180^0$
$r = 900 – ip$
From Snell's law,

$$\frac{sini_p}{sinr} = \mu$$

where μ is the refractive index of the medium (glass)
Substituting for *r*, we get

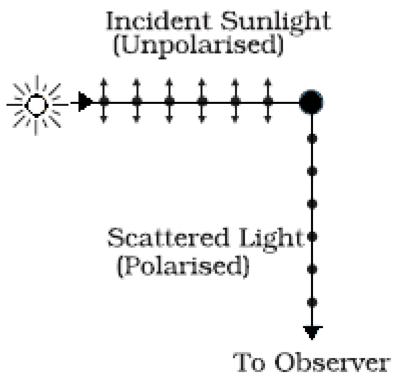
$$\frac{sini_p}{sin(90 - i_p)} = \frac{sini_p}{cosi_p} = \mu$$

$$\therefore \tan i_p = \mu$$

Tangent of polarizing angle is numerically equal to refractive index of medium

Polarisation by scattering

The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a Polaroid which is rotated. This is nothing but sunlight, which has changed its direction (having been scattered) on encountering the molecules of the earth's atmosphere



As shown in figure, the incident sunlight is unpolarised. The dots stand for polarisation perpendicular to the plane of the figure. The double arrows show polarisation in the plane of the figure. There is no phase relation between these two in unpolarised light.
Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions.
We have drawn an observer looking at 90° to the direction of the sun. Clearly, charges accelerating parallel to the double arrows do not radiate energy towards this observer since their acceleration has no transverse component. The radiation scattered by the molecule is therefore represented by dots. It is polarized perpendicular to the plane of the figure. This explains the polarization of scattered light from the sky.

Fresnel distance, ray optics is a limiting case of wave optics

Fresnel distance is that distance from the slit at which the spreading of light due to diffraction becomes equal to the size of the slit. It is generally denoted by Z_F
We know that the first secondary minimum is formed at an angle θ_1 such that

$$\theta_1 = \frac{\lambda}{d}$$

After travelling a distance D, the width acquired by the beam due to diffraction is $D\lambda/d$
At Fresenel distance Z_F

$$\frac{Z_F \lambda}{d} = d$$

$$Z_F = \frac{d^2}{\lambda}$$

If the distance D between the slit and the screen is less than Fresnel distance Z_F then the diffraction effects may be regarded as absent. So, ray optics may be regarded as limiting case of wave optics

Solved Numerical

Light of wave length 600nm is incident on an aperture of size 2mm. calculate the distance upto which the ray of light can travel such that its spread is less than the size of the aperture
Solution

$$Z_F = \frac{d^2}{\lambda} = \frac{(2 \times 10^{-3})^2}{600 \times 10^{-9}} = 6.67\ m$$

Doppler effect for light

If there is no medium and the source moves away from the observer, then later wavefronts have to travel a greater distance to reach the observer and hence take a longer time. The time taken between the arrival of two successive wavefronts is hence longer at the observer than it is at the source.
Thus, when the source moves away from the observer the frequency as measured by the source will be smaller. This is known as the *Doppler effect*.
Astronomers call the increase in wavelength due to doppler effect as *red shift* since a wavelength in the middle of the visible region of the spectrum moves towards the red end of the spectrum.
When waves are received from a source moving towards the observer, there is an apparent decrease in wavelength, this is referred to as *blue shift.*

For velocities small compared to the speed of light, we can use the same formulae which we use for sound waves. The fractional change in frequency Δv/v is given by $-V_{radial}/c$, where V_{radial} is the component of the source velocity along the line joining the observer to the source relative to the observer; V_{radial} is considered positive when the source moves away from the observer. Thus, the Doppler shift can be expressed as:

$$\frac{\Delta v}{v} = -\frac{V_{radial}}{c}$$

Solved numerical

Q) Certain characteristic wavelengths of the light from a galaxy in the constellation Virgo are observed to be increased in wave length, as compaired with terrestrial sources, by 0.4%. What is the radial speed of this galaxy with respect to the earth? Is it approaching or receding?

<u>Solution</u>

From formula

$$\frac{\Delta v}{v} = -\frac{V_{radial}}{c}$$

We know that

$$\frac{\Delta v}{v} = -\frac{\Delta\lambda}{\lambda}$$

Thus

$$\frac{\Delta\lambda}{\lambda} = \frac{V_{radial}}{c}$$

Given : Δλ/λ = 0.004

$$V_{radial} = \frac{\Delta\lambda}{\lambda} c$$

$$V_{radial} = 0.004 \times 3 \times 10^8 = 1.2 \times 10^6\ ms^{-1}$$

Since v_{radial} is positive therefore galaxy is receding

Q) the red shift of radiation from a distant nebula consists of the light known to have a wavelength 4340×10^{-8} cm when observed in laboratory, appearing to have a wavelength of 4362×10^{-8} cm. What is the speed of the nebula in the line of sight relative to the earth ? Is it approaching or receding

Solution:

$\Delta\lambda = 4362\times10^{-8} - 4340\times10^{-8} = 22\times10^{-8}$ cm thus

$\Delta\lambda/\lambda = 22\times10^{-8}/4340\times10^{-8} = 0.0004$

Thus $\Delta\nu/\nu = -0.0004$

$$\frac{\Delta v}{v} = -\frac{V_{radial}}{c}$$

$$-0.0004 = -\frac{V_{radial}}{3\times10^{8}}$$

$V_{radial} = (0.0004)(3\times10^{8}) = 0.12852\times10^{8} = 1.2\times10^{5}$ m/s

Since V_{radial} is positive nebula is receding

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